Refinement of the acceleration approach for GRACE gravity field recovery

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Introduction:

For any gravity field mission based on the low-low satellite-to-satellite (llSST) concept the inter-satellite observations needs to be connected to the gravity field. The so-called acceleration approach is one such possibility which connects range accelerations and velocity differences to difference of the gradient of the gravitational potential ∇V_{AB}.

The lower precision of the velocity differences (GPS derived) requires to reduce the approach to residual quantities using an *a priori* gravity field which allows subsequently to neglect the residual velocity difference term. We present two ways to handle the residual velocity difference term: (1) based on rotations and (2) a rigorous modelling.

Acceleration approach:

Several variant of the acceleration approach exist. Here we refer to as:

Range: $\vec{x}_{AB} = \rho\,\vec{e}^{a}_{AB}$

Range rate: $\dot{\vec{x}}_{AB} = \dot{\rho}\,\vec{e}^{a}_{AB} + \rho\,\dot{\vec{e}}^{a}_{AB}$

Range acceleration: $\ddot{\vec{x}}_{AB} = \ddot{\rho}\,\vec{e}^{a}_{AB} + 2\,\dot{\rho}\,\dot{\vec{e}}^{a}_{AB} + \rho\,\ddot{\vec{e}}^{a}_{AB}$

Multiplication with unit vectors in along-track (a), cross-track (c) and radial direction (r) yields the acceleration approach for the llSST case:

$$\begin{array}{lllll} \ddot{\vec{x}}_{AB}\cdot\vec{e}^{a}_{AB} = \ddot{\rho} & + & 0 & + & \rho\,\ddot{\vec{e}}^{a}_{AB}\cdot\vec{e}^{a}_{AB} \\ \ddot{\vec{x}}_{AB}\cdot\vec{e}^{c}_{AB} = 0 & + & 0 & + & \rho\,\ddot{\vec{e}}^{a}_{AB}\cdot\vec{e}^{c}_{AB} \\ \ddot{\vec{x}}_{AB}\cdot\vec{e}^{r}_{AB} = 0 & + & 2\,\dot{\rho}\,\|\dot{\vec{e}}^{a}_{AB}\| & + & \rho\,\ddot{\vec{e}}^{a}_{AB}\cdot\vec{e}^{r}_{AB} \end{array}$$

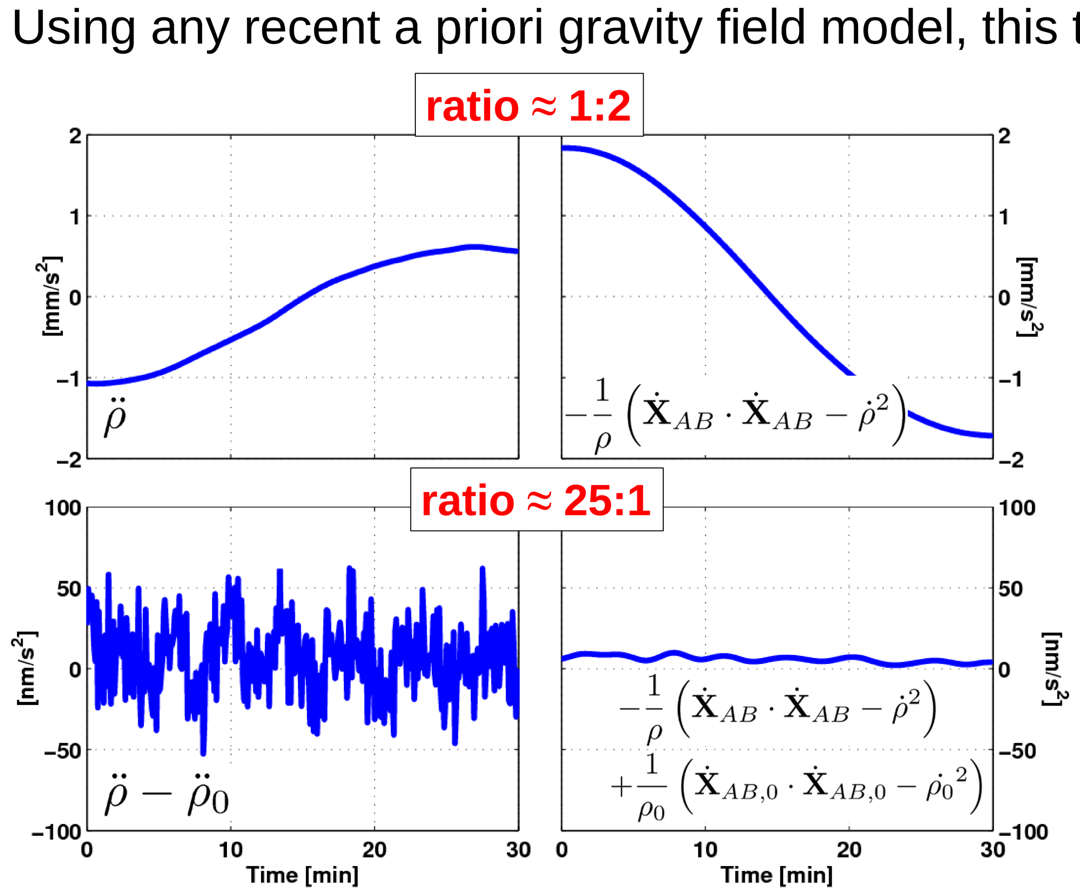
llSST case (first line)

Standard approach:

The second term of the llSST case can be rewritten:

$$\ddot{\vec{x}}_{AB}\cdot\vec{e}^{a}_{AB} = \ddot{\rho} - \frac{1}{\rho}\left(\dot{\vec{X}}_{AB}\cdot\dot{\vec{X}}_{AB} - \dot{\rho}^{2}\right)$$

The lower precision of $\dot{\vec{X}}_{AB}$ requires the reduction to a residual quantity. Using any recent a priori gravity field model, this term can be approximated sufficiently well and the residual range acceleration term dominates. The approximation is sufficient for GRACE and may be improved by iteration.



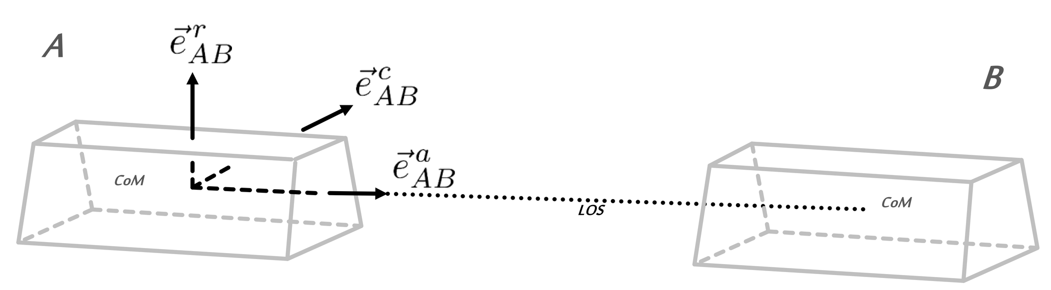
For future concepts offering higher precisions, the number of iterations may become intolerable and alternative observables may allow for new possibilities.

Alternative 1: rotational approach

One alternative is to express the acceleration approach in the so-called instantaneous rotation reference frame (IRRF) which is defined epoch-wise by the along-track, cross-track and radial unit vector.

Definition:

$$R^{I}_{F} = \begin{pmatrix} (\vec{e}^{a}_{AB})^{T} \\ (\vec{e}^{c}_{AB})^{T} \\ (\vec{e}^{r}_{AB})^{T} \end{pmatrix}$$



The IRRF is a moving reference frame and therefore "moving-frame-quantities" are required:

$$\begin{aligned} R^{I}_{F}\vec{e}^{\,a,c,r}_{AB} &= \vec{e}^{\,a,c,r}_{AB,F} \\ R^{I}_{F}\dot{\vec{e}}^{a}_{AB} &= \dot{\vec{e}}^{a}_{AB,F} + \vec{\omega}\times\vec{e}^{a}_{AB,F} = \vec{\omega}\times\vec{e}^{a}_{AB,F} \\ R^{I}_{F}\ddot{\vec{e}}^{a}_{AB} &= \ddot{\vec{e}}^{a}_{AB,F} + 2\vec{\omega}\times\dot{\vec{e}}^{a}_{AB,F} + \vec{\omega}\times\left(\vec{\omega}\times\vec{e}^{a}_{AB,F}\right) + \dot{\vec{\omega}}\times\vec{e}^{a}_{AB,F} \\ &= \vec{\omega}\times\left(\vec{\omega}\times\vec{e}^{a}_{AB,F}\right) + \dot{\vec{\omega}}\times\vec{e}^{a}_{AB,F} \end{aligned}$$

with $\dot{\vec{e}}^{\,a,c,r}_{AB,F} = \ddot{\vec{e}}^{\,a,c,r}_{AB,F} = \vec{0}$

and $\Omega = R^{I}_{F}\left(\dot{R}^{I}_{F}\right)^{T} = \begin{pmatrix} 0 & -\omega^{r} & \omega^{c} \\ \omega^{r} & 0 & -\omega^{a} \\ -\omega^{c} & \omega^{a} & 0 \end{pmatrix}$ $\quad \dot{R}^{I}_{F} = \begin{pmatrix} (\dot{\vec{e}}^{a}_{AB})^{T} \\ (\dot{\vec{e}}^{c}_{AB})^{T} \\ (\dot{\vec{e}}^{r}_{AB})^{T} \end{pmatrix}$

Using the following realization for the IRRF

$$\vec{e}^{a}_{AB} = \frac{\vec{x}_{AB}}{\|\vec{x}_{AB}\|} \quad \vec{e}^{v}_{AB} = \frac{\dot{\vec{x}}_{AB}}{\|\dot{\vec{x}}_{AB}\|} \quad \vec{e}^{c}_{AB} = \frac{\vec{e}^{v}_{AB}\times\vec{e}^{a}_{AB}}{\|\vec{e}^{v}_{AB}\times\vec{e}^{a}_{AB}\|} \quad \vec{e}^{r}_{AB} = \frac{\vec{e}^{a}_{AB}\times\vec{e}^{c}_{AB}}{\|\vec{e}^{a}_{AB}\times\vec{e}^{c}_{AB}\|}$$

yields:

$$\begin{array}{llll} \nabla V_{AB,F}\cdot\vec{e}^{a}_{AB,F} = & \ddot{\rho} & -\rho\left(\omega^{c}\right)^{2} \\ \nabla V_{AB,F}\cdot\vec{e}^{c}_{AB,F} = & & -\rho\,\omega^{a}\omega^{c} \\ \nabla V_{AB,F}\cdot\vec{e}^{r}_{AB,F} = & -2\,\dot{\rho}\,\omega^{c} & -\rho\,\dot{\omega}^{c} \end{array}$$

Discussion:

- Acceleration approach can be expressed by rotations.
- Observation of ω^{c} and $\dot{\omega}^{c}$ allows for a second (observed) component. Observation of ω^{a} for a third (observed) direction.
- Necessity for GPS observations remains (determination of the IRRF)
- Bandikova et al. (2013) showed that the precision of the from the GRACE star cameras derived the angular rates is insufficient for the approach to be applicable at the moment.
- The LRI in GRACE Follow-On offers to observe angular rates with higher precision: tests pending.

Alternative 2: rigorous approach

The second approach is to consider the residual velocity differences as unknown but estimable as they depend on the unknown residual gravity field parameters and initial conditions.

$$\ddot{\rho} = \ddot{\vec{x}}_{AB}\cdot\vec{e}^{a}_{AB} + \frac{1}{\rho}\left(\dot{\vec{X}}_{AB}\cdot\dot{\vec{X}}_{AB} - \dot{\rho}^{2}\right)$$

Introducing a priori values and linearization yields:

$$\begin{aligned} \ddot{\rho} - \ddot{\rho}_{0} \;=\; & \Delta\nabla V_{AB}\cdot\vec{e}^{a}_{AB} + \ldots \\ & + \frac{\partial}{\partial\bar{C}_{lm}}\left[\frac{1}{\rho}\left(\dot{\vec{X}}_{AB}\cdot\dot{\vec{X}}_{AB} - \dot{\rho}^{2}\right)\right] \\ & + \frac{\partial}{\partial\bar{S}_{lm}}\left[\frac{1}{\rho}\left(\dot{\vec{X}}_{AB}\cdot\dot{\vec{X}}_{AB} - \dot{\rho}^{2}\right)\right] \\ & + \frac{\partial}{\partial\vec{s}^{A}_{0}}\left[\frac{1}{\rho}\left(\dot{\vec{X}}_{AB}\cdot\dot{\vec{X}}_{AB} - \dot{\rho}^{2}\right)\right] \\ & + \frac{\partial}{\partial\vec{s}^{B}_{0}}\left[\frac{1}{\rho}\left(\dot{\vec{X}}_{AB}\cdot\dot{\vec{X}}_{AB} - \dot{\rho}^{2}\right)\right] \end{aligned}$$

Discussion:

- Approach is more rigorous in the sense that (1) it is a better approximation than the standard approach and (2) orbit and parameter estimation are not separated, cf. Meyer et al. (2014).
- Approach requires the partial derivatives towards the unknown spherical harmonic coefficients and the initial conditions of the orbits of Grace A and Grace B.
- The partial derivatives can only partially be determined analytically, i.e. variational equations need be solved in parallel.

References

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