Refinement of the acceleration approach for GRACE gravity field recovery
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Introduction:
For any gravity field mission based on the low-low satellite-to-satellite (LLSST) concept the inter-satellite observations needs to be connected to the gravity field. The so-called acceleration approach is one such possibility which connects range accelerations and velocity differences to difference of the gradient of the gravitational potential $\nabla V_{AB}$.

The lower precision of the velocity differences (GPS derived) requires to reduce the approach to residual quantities using an a priori gravity field which allows subsequently to neglect the residual velocity difference term. We present two ways to handle the residual velocity difference term: (1) based on rotations and (2) a rigorous modelling.

Acceleration approach:
Several variant of the acceleration approach exist. Here we refer to as:

**Range acceleration:**

$$\ddot{\Delta} x_{AB} = \rho \left( \frac{\partial^2 V_{AB}}{\partial t^2} \right)$$

**Range rate:**

$$\dot{\Delta} v_{AB} = \rho \left( \frac{\partial V_{AB}}{\partial t} \right)$$

Multiplication with unit vectors in along-track (a), cross-track (c) and radial direction (r) yields the acceleration approach for the LLSST case:

$$\ddot{\Delta} x_{AB} = \rho \left( \frac{\partial^2 V_{AB}}{\partial t^2} \right)$$

The lower precision $\approx 1:2$ yields the acceleration approach for the LLSST case:

$$\newcommand{\Delta}{\\Delta x_{AB}}$$

$$\Delta x_{AB} = \rho \left( \frac{\partial^2 V_{AB}}{\partial t^2} \right)$$

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Alternative 1: rotational approach

One alternative is to express the acceleration approach in the so-called instantaneous rotation reference frame (IRRF) which is defined epoch-wise by the along-track, cross-track and radial unit vector.

**Definition:**

$$R^T_1 \left( \frac{\partial V_{AB}}{\partial \Delta t} \right) = \left( \begin{array}{ccc} 0 & -\omega^c & \omega^a \\ \omega^c & 0 & -\omega^r \\ -\omega^a & \omega^r & 0 \end{array} \right) \left( \begin{array}{c} \dot{\Delta} x_{AB} \\ \dot{\Delta} v_{AB} \\ \Delta \omega_{AB} \end{array} \right)$$

The IRRF is a moving reference frame and therefore “moving-frame-quantities” are required:

$$R^T_1 \left( \frac{\partial V_{AB}}{\partial \Delta t} \right) = \left( \begin{array}{ccc} 0 & -\omega^c & \omega^a \\ \omega^c & 0 & -\omega^r \\ -\omega^a & \omega^r & 0 \end{array} \right) \left( \begin{array}{c} \dot{\Delta} x_{AB} \\ \dot{\Delta} v_{AB} \\ \Delta \omega_{AB} \end{array} \right)$$

Using the following realization for the IRRF

$$\Delta x_{AB} = \frac{\dot{\Delta} x_{AB} - \ddot{\Delta} x_{AB} \cdot \Delta t}{2}$$

yields:

$$\nabla V_{AB} \cdot \Delta \omega_{AB} = \rho \left( \dot{\Delta} x_{AB} - \ddot{\Delta} x_{AB} \cdot \Delta t \right)$$

Discussion:

- Acceleration approach can be expressed by rotations.
- Observation of $\omega^r$ and $\omega^c$ allows for a second (observed) component. Observation of $\omega^a$ for a third (observed) direction.
- Necessity for GPS observations remains (determination of the IRRF)
- Bandikova et al. (2013) showed that the precision of the from the GRACE star camera derived the angular rates is insufficient for the approach to be applicable at the moment.
- The LRI in GRACE Follow-On offers to observe angular rates with higher precision: tests pending.

Alternative 2: rigorous approach

The second approach is to consider the residual velocity differences as unknown but estimable as they depend on the unknown residual gravity field parameters and initial conditions.

$$\dot{\Delta} \omega_{AB} = \frac{\Delta \Delta \omega_{AB}}{\Delta \omega_{AB}}$$

Introducing a priori values and linearization yields:

$$\dot{\Delta} \omega_{AB} = \frac{\Delta \Delta \omega_{AB}}{\Delta \omega_{AB}}$$

Discussion:

- Approach is more rigorous in the sense that (1) it is a better approximation than the standard approach and (2) orbit and parameter estimation are not separated, cf. Meyer et al. (2014).
- Approach requires the partial derivatives towards the unknown spherical harmonic coefficients and the initial conditions of the orbits of Grace A and Grace B.
- The partial derivatives can only partially be determined analytically, i.e. variational equations need be solved in parallel.

References


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