A wavelet-based non-stationary noise modeling in GRACE gravity field determination

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Introduction

The assumption of noise stationarity in the temporal gravity field determination using GRACE range rate observations, may lead to the loss of some dynamic features of the data and thus result in inaccuracy in gravity parameter estimation. In this work, a wavelet-based general least squares is used for parameter estimation using GRACE observations with a non-stationary fractional noise model. After both observation and design matrices have been transformed to wavelet domain, the dense variance-covariance matrix of the noise is diagonalized by exploiting the whitening (or Karhunen-Loève type) property of the Discrete Wavelet Transform (DWT). Implementation of gravity parameter estimation in wavelet domain, estimation of the noise covariance using the residual coefficients and comparison of the results with the ITSG-2016 model are discussed.

Methodology

We study multilevel signal decomposition methods on a wavelet orthonormal basis. The Discrete Wavelet Transform (DWT) provides a compact representation of a signal in time and frequency that can be computed efficiently. The DWT analysis can be performed using a fast, pyramidal algorithm related to a filter banks. In the pyramidal algorithm, the signal is analyzed at different frequency bands with different resolution by decomposing into approximation and detail information. The approximation is then further decomposed using the same wavelet decomposition step (Figure 1). In our implementation, the 20 coefficient wavelet family proposed by Daubechies [1] with 8 level of decomposition is used.

Applying the Daubechies wavelet transform to observations and design matrix, we will have a general linear model in the wavelet domain as follows:

$$\boldsymbol{l}^W = \boldsymbol{A}^W \boldsymbol{x} + \boldsymbol{e}^W,$$

where $\boldsymbol{l}^W$ and $\boldsymbol{e}^W$ are the results of applying DWT respectively to the observations and residuals up to level J and $\boldsymbol{A}^W$ is the wavelet transform applying to each columns of the design matrix $\boldsymbol{A}$.

Looking through the power spectral density of the residuals, one can approximate the behavior of the noise in low-frequency regions with 1/f-processes. One well-known model of the 1/f-like processes is the fractional Brownian motion (fBm). It is a zero mean, non-stationary, and nondifferentiable process [2] with the spectral density of:

$$\lim_{f\to 0} S(f) \approx \frac{\sigma^2}{f^{2H-1}},$$

characterized by a single scalar self-similarity parameter H, called the Hurst exponent. The orthonormal wavelet decomposition acts as a Karhunen-Loève expansion of these type of processes [3]. Therefore, as an acceptable approximation, the variance-covariance matrix of the noise is diagonalized by the wavelet transform [2] to take the following form ,

$$\boldsymbol{\Sigma} = diag\left\{\{S_{a_J}\},\{S_{d_J}\},\{S_{d_{J-1}}\},\ldots,\{S_{d_1}\}\right\},$$

Where,

$$S_{a_J} = \frac{\sigma^2 c_H (2^{J+1})^{2H-1}}{2(2\pi)^{2H-1}(1-H)},$$

$$S_{d_j} = \frac{\sigma^2 c_H (2^j)^{2H-1}(1-2^{2H-2})}{(2\pi)^{2H-1}(1-H)}, \text{ for j} = 1,\ldots,\text{J}.$$

This means, the variances of the approximation and detail coefficients of each level are dependent on their level of decomposition.

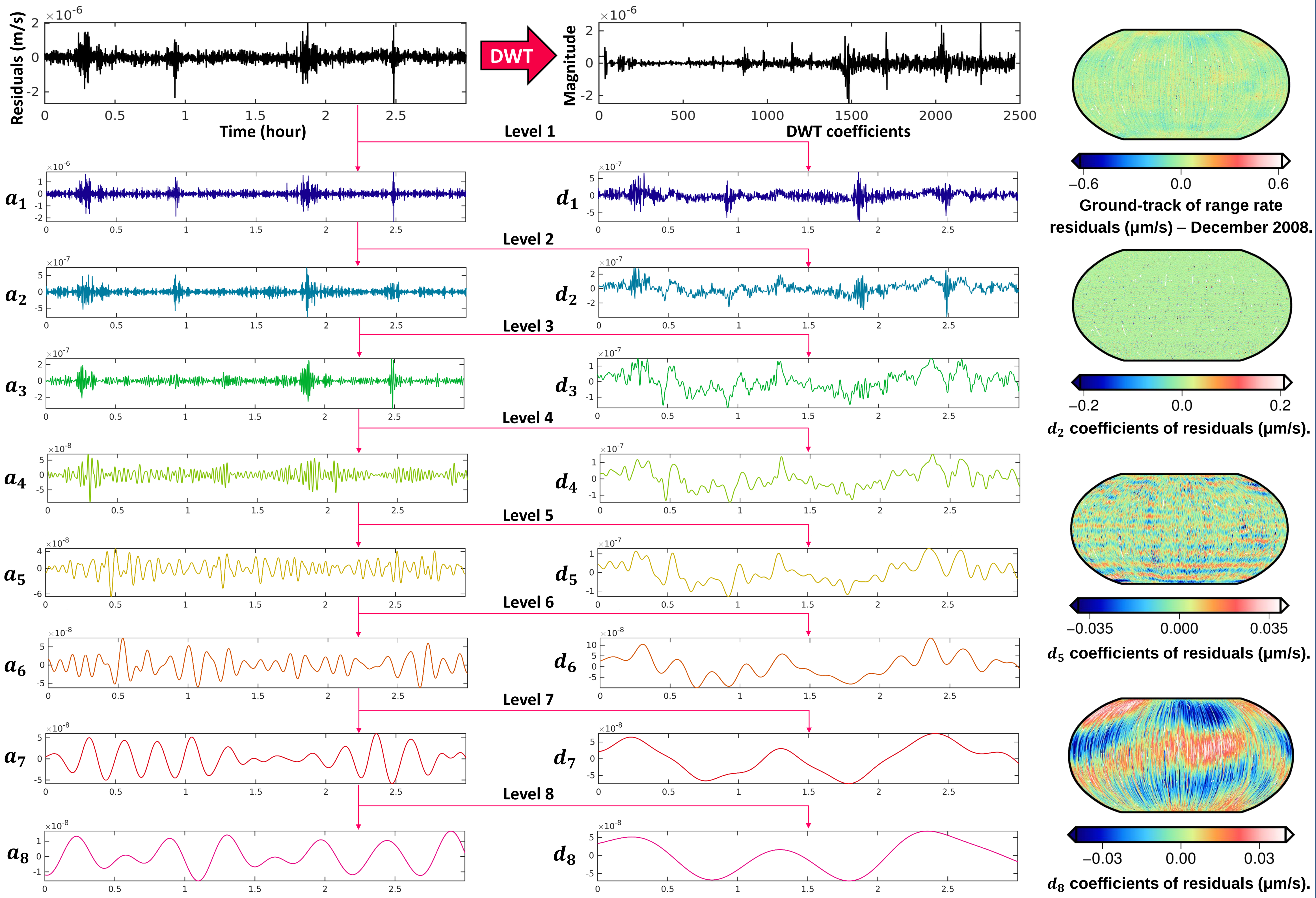

Figure 1. 8-level Daubechies decomposition of range rate residuals with the ground-track of corresponding time series of selected detail coefficients.

Results

Based on the ITSG-GRACE2016 processing scheme, the gravity field solution using Wavelet-based General Least Squares (WGLS) with maximum spherical harmonic expansion degree of 60 is computed for the test month (December 2008). In the figure 2, the degree variances of the solution and the corresponding ITSG and CSR monthly solutions are compared to the GOCO05s model. To compare the solutions in frequency domain, the spectrograms of the range rate residuals from the ITSG solution and the wavelet-based method are shown (Figure 3).

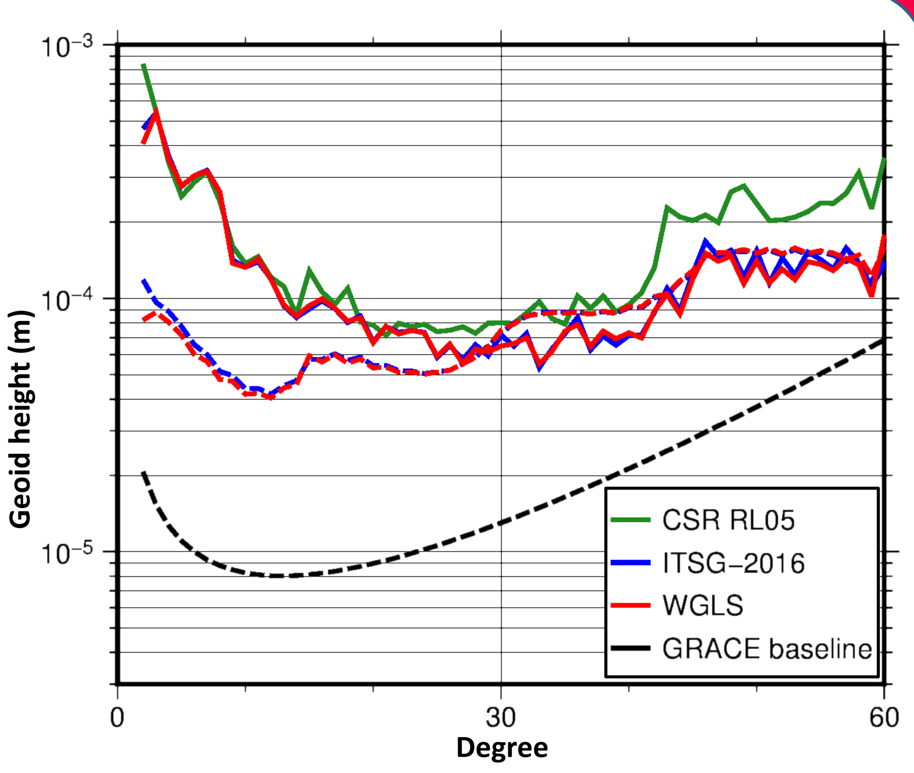

Figure 2. Degree variances of gravity field solutions - December 2008.

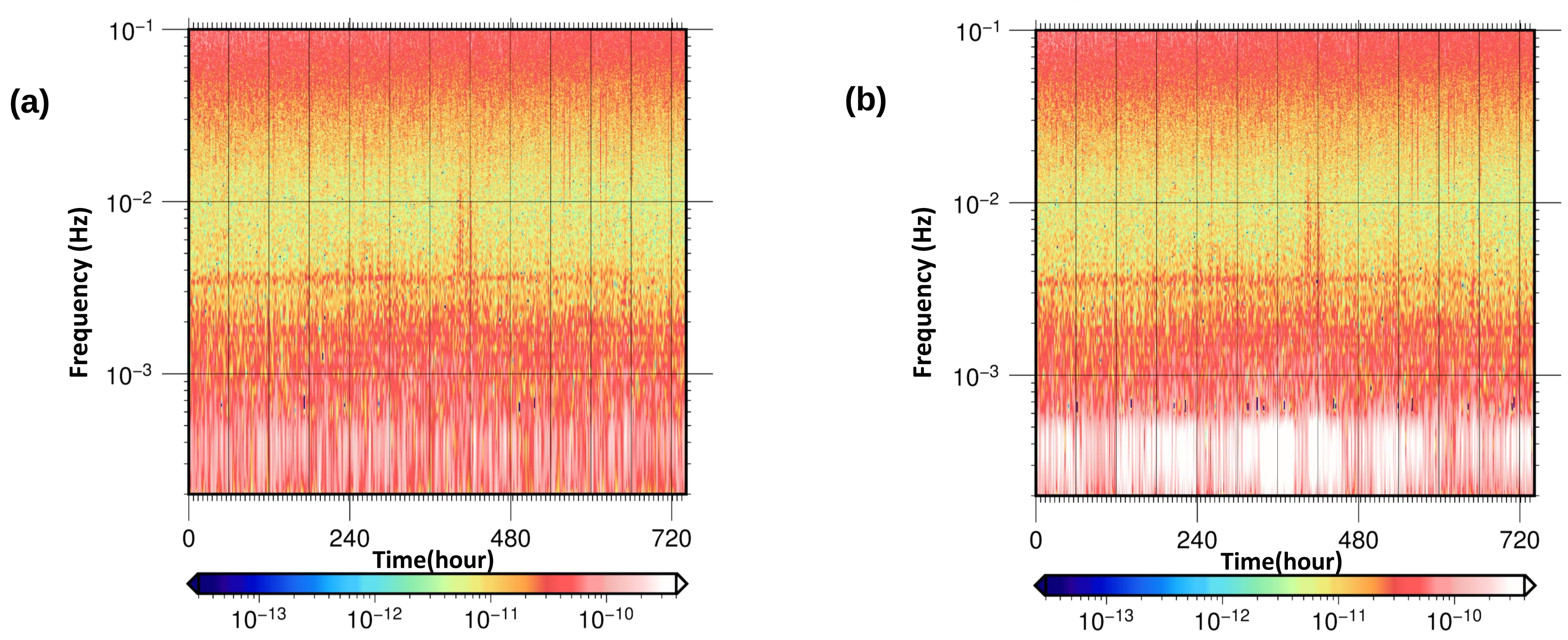

Figure 3. Comparison of the range rate residuals spectrogram (a) ITSG , (b) WGLS.

Discussion and Conclusion

To sum up, using wavelet decomposition for range rate residuals, gives us a better understanding of noise behavior than the Fourier analysis, as the traditional spectral analysis is unable to provide any information about the time dependency of the frequency contents. In GRACE range rate observations, the noise is inherently time-varying, therefore a general model to incorporate the noise stochastic properties is needed for a more accurate data analysis.

Performing the parameter estimation in wavelet domain with a non-stationary fractional noise model shows improvement in higher degree coefficients in degree variances plot (Figure 2), comparing to the ITSG-2016 solution of the test month. As the method only considers low-frequency model for the noise, it could be refined in respect to a complete modeling of the noise behavior. Future research will concentrate on the physical understanding of the decomposed coefficients and implementation of a more general noise model.

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