CNES/GRGS solutions
Focus on the inversion process

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(2) GET/UMR5563/OMP/GRGS, Toulouse, France
(3) Géode & Cie, Toulouse, France

Overview

- CNES/GRGS solutions
  - GRACE data until degree and order 80
  - LAGEOS 1&2 + STARLETTE + STELLA until 30
  - Available as monthly and 10-day solutions
  - No need for filtering

Summary

- Focus of the presentation: inversion process

- Objectives
  - Shed a light on what we do, and why
  - Why we use truncated SVD instead of a-posteriori filtering (two-step process)
  - Increase interest in our solutions

Inversion process

- **<u>Least squares: the solution to all problems?</u>**

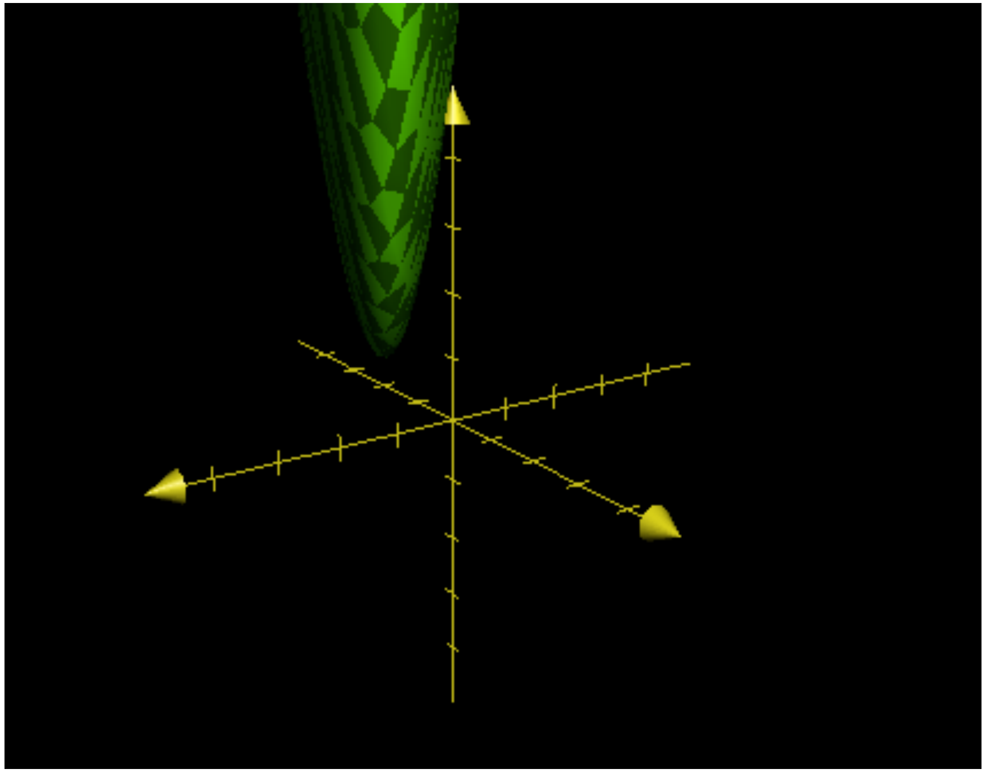
- Standard method = least squares + destriping

- $S(p) = sum(|y_i - f_i(p)|^2)$

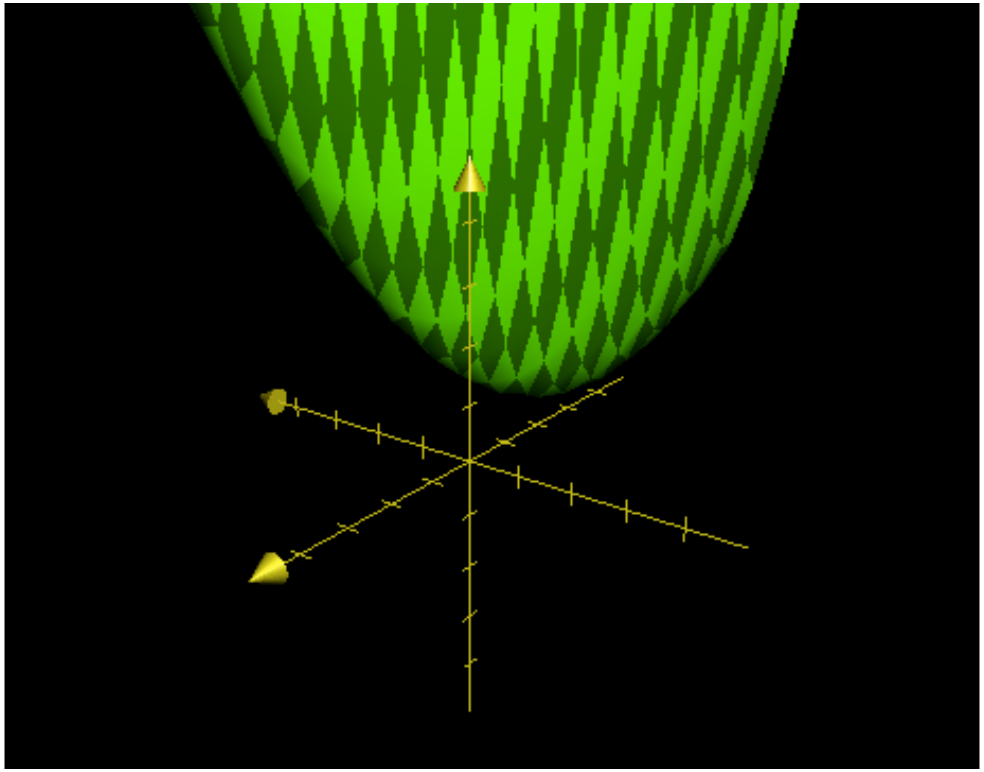
- S is a quadratic function, a « n-dimension paraboloid ». It has a minimum.

Inversion process

❖ S(p) with a two dimensional parameter p=(x,y)



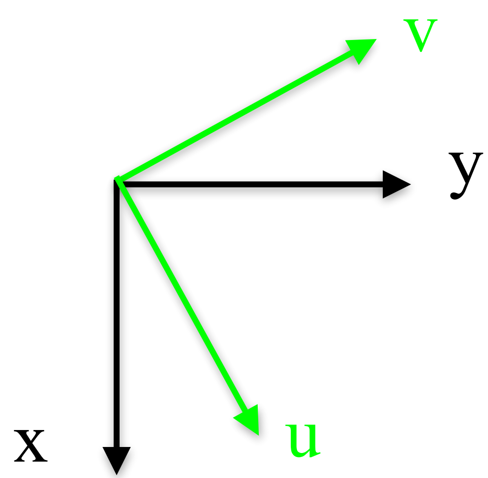
Front view



Side view

Inversion process

- Canonical basis (x,y) Paraboloid axes (u,v)

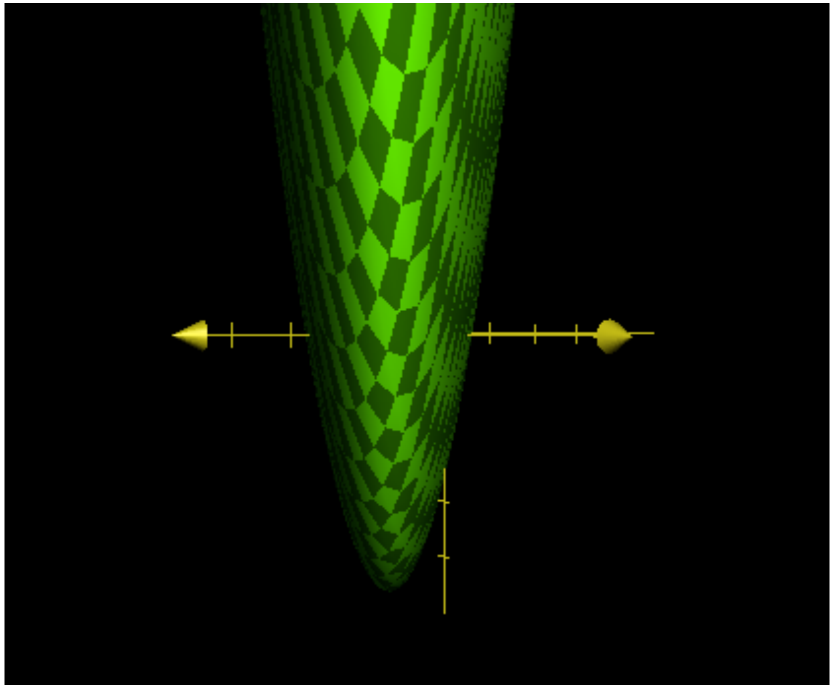


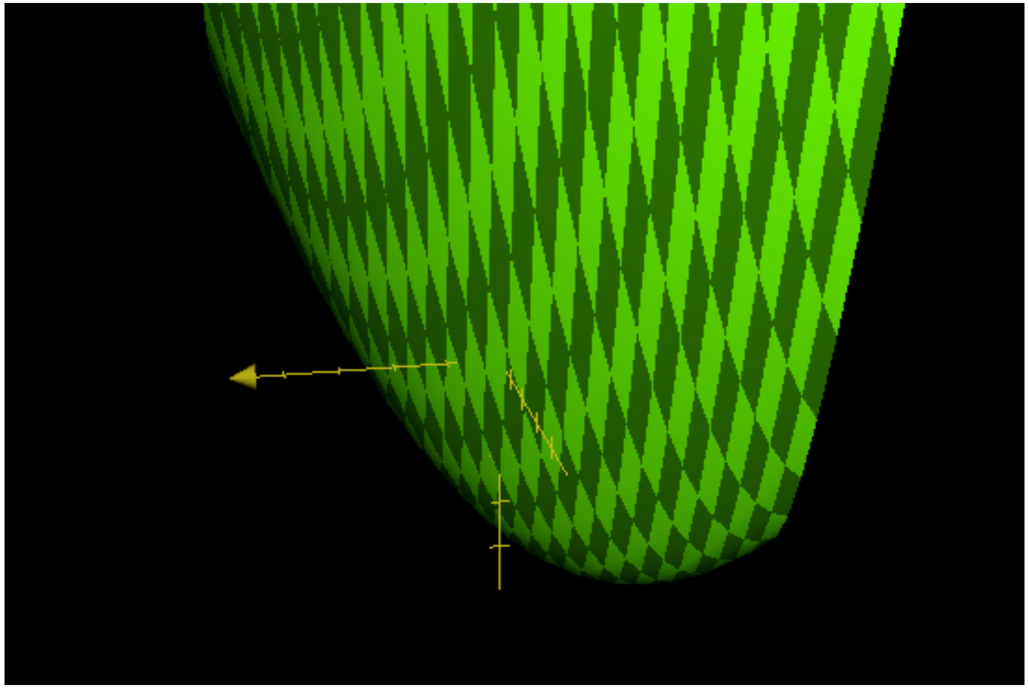
- Least squares solution: corrections on the x and y axes to reach the minimum: $p_{min} = p_0 + corr_x\, x + corr_y\, y$

Inversion process

- Challenge: suppose you're allowed one single move on one axis…

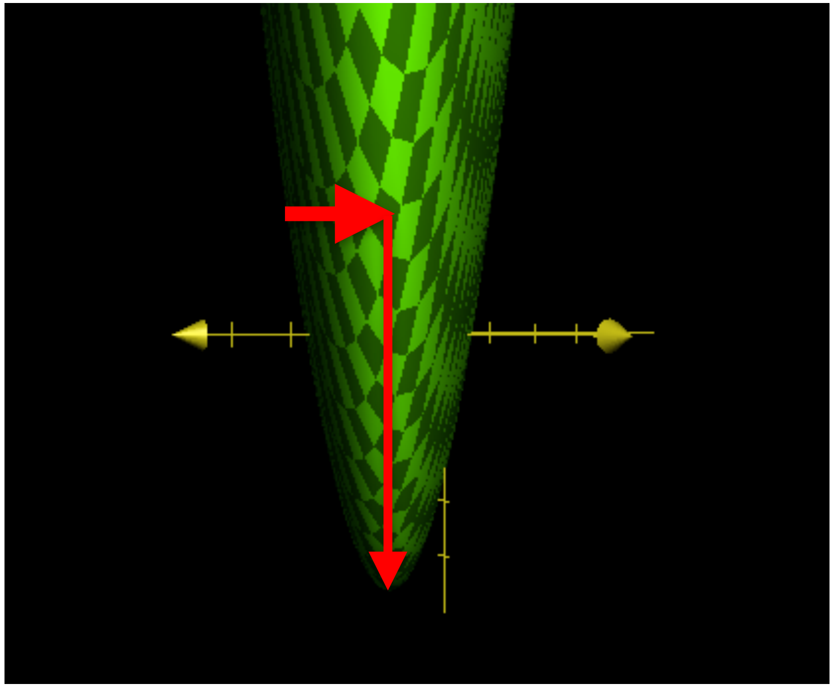
  … which one would you choose? x? y? u? v?

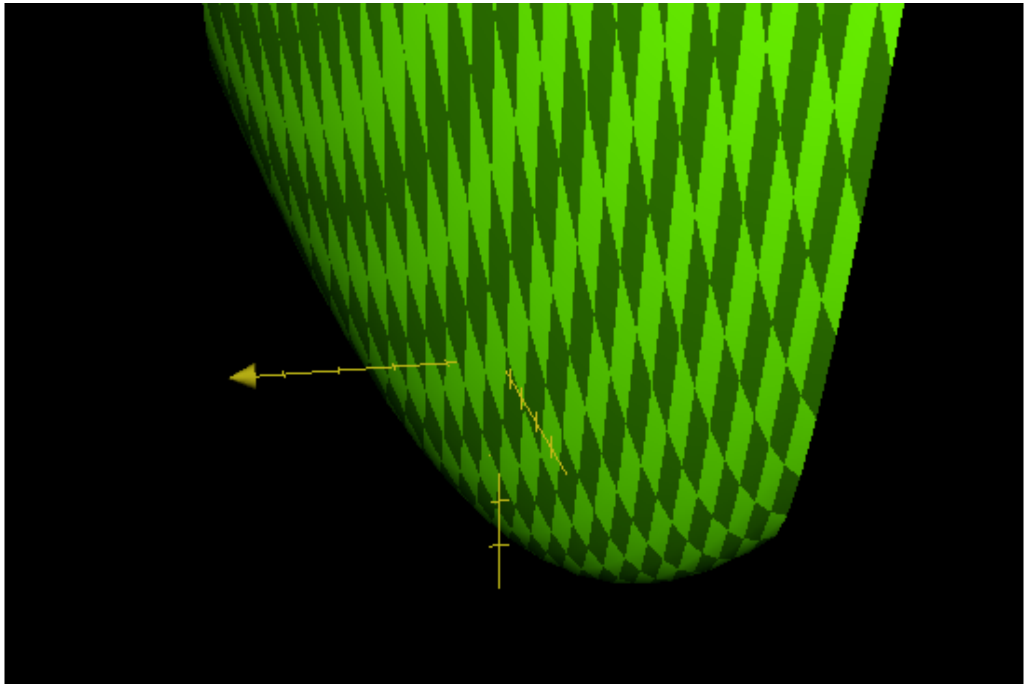




Inversion process

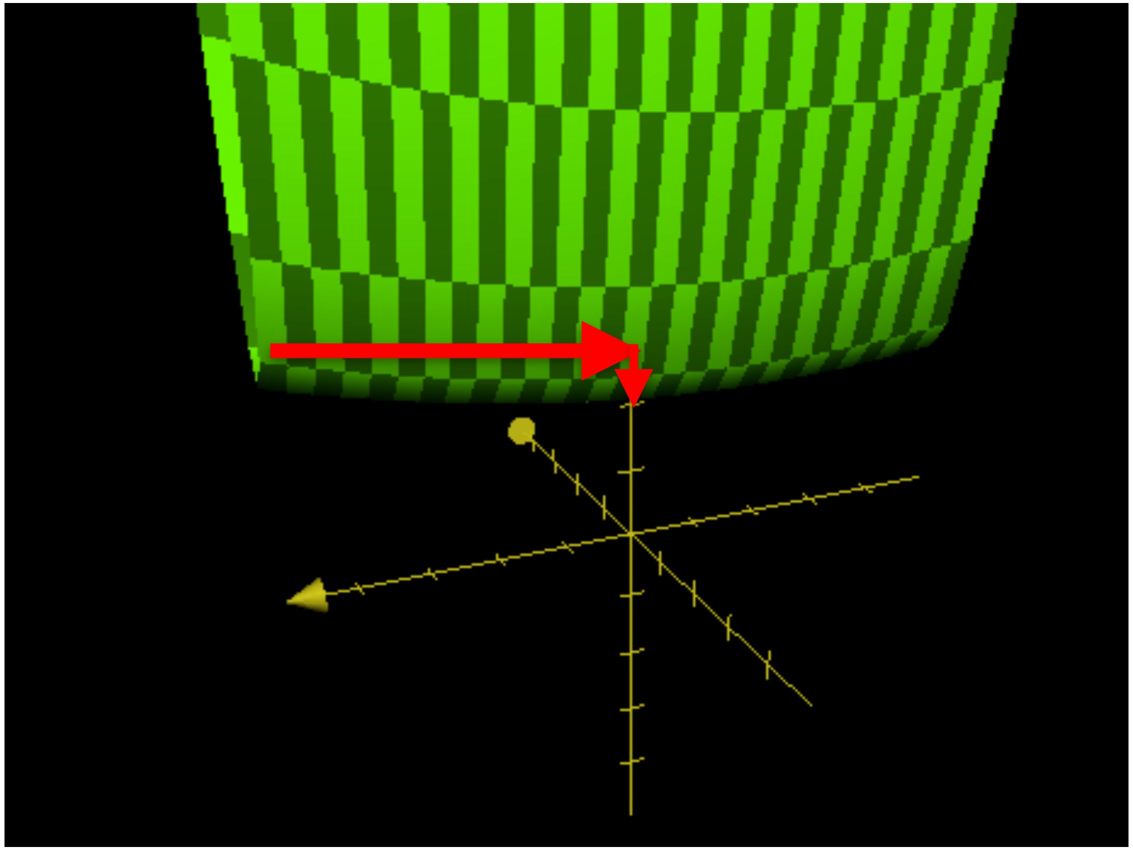
- The best result for your effort: in the direction of the steepest axis of the paraboloid
- Little horizontal move (correction on the parameter) with big vertical move (improvements on the residuals)





Inversion process

❖ What happens with a very flat axis?



❖ Very big correction (horizontal) for little improvement in residuals (vertical)

Inversion process

- Is it really worth the effort?

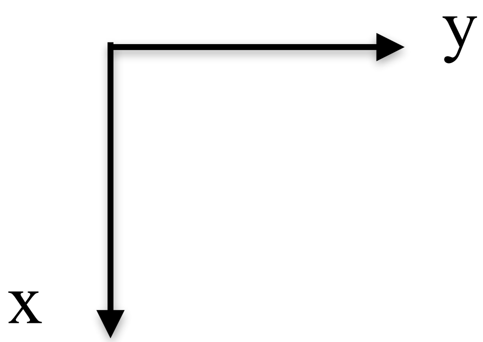
- Risk to leave the linear approximation validity zone

- Is may NOT always be relevant to look for the minimum in EVERY direction

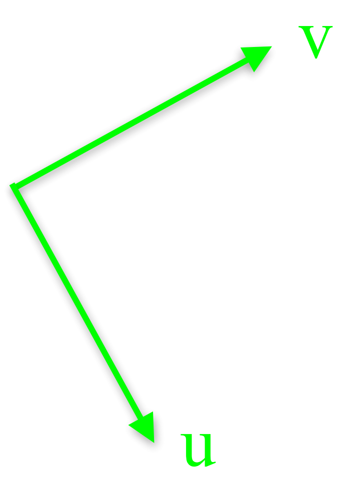
- The least squares solution may just simply not be the best.

- Let's have a look at the spherical harmonics

- A) Canonical basis (x,y)

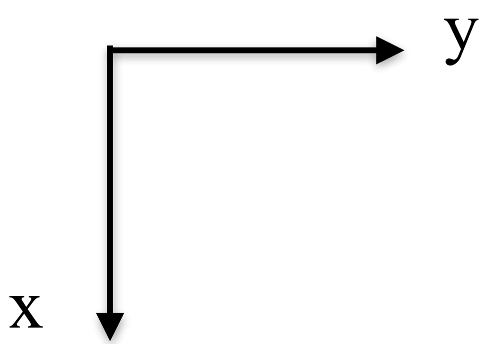


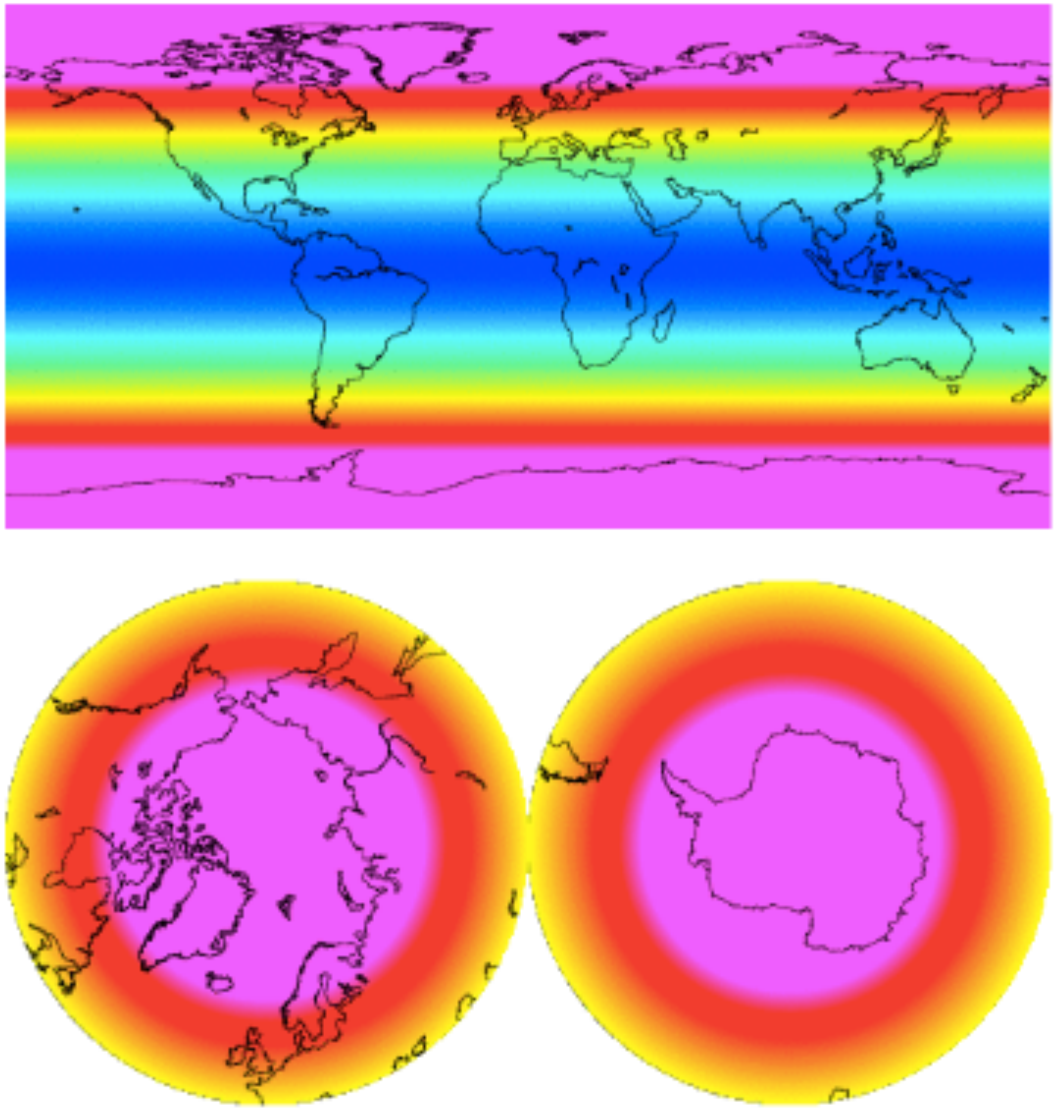
B) Paraboloid axes (u,v), i.e. eigenvectors of the normal matrix



Inversion process

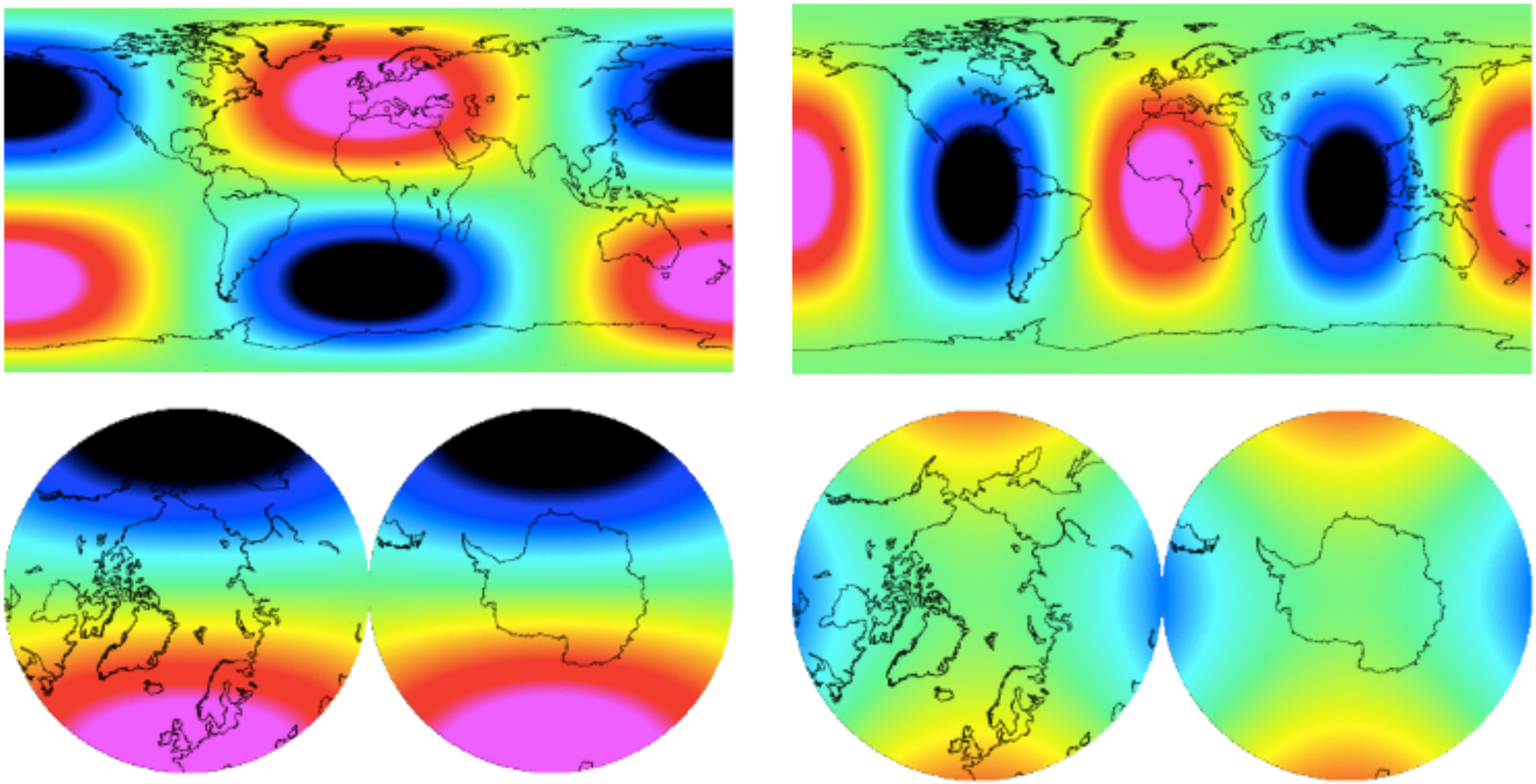
❖ A) Canonical basis: C20





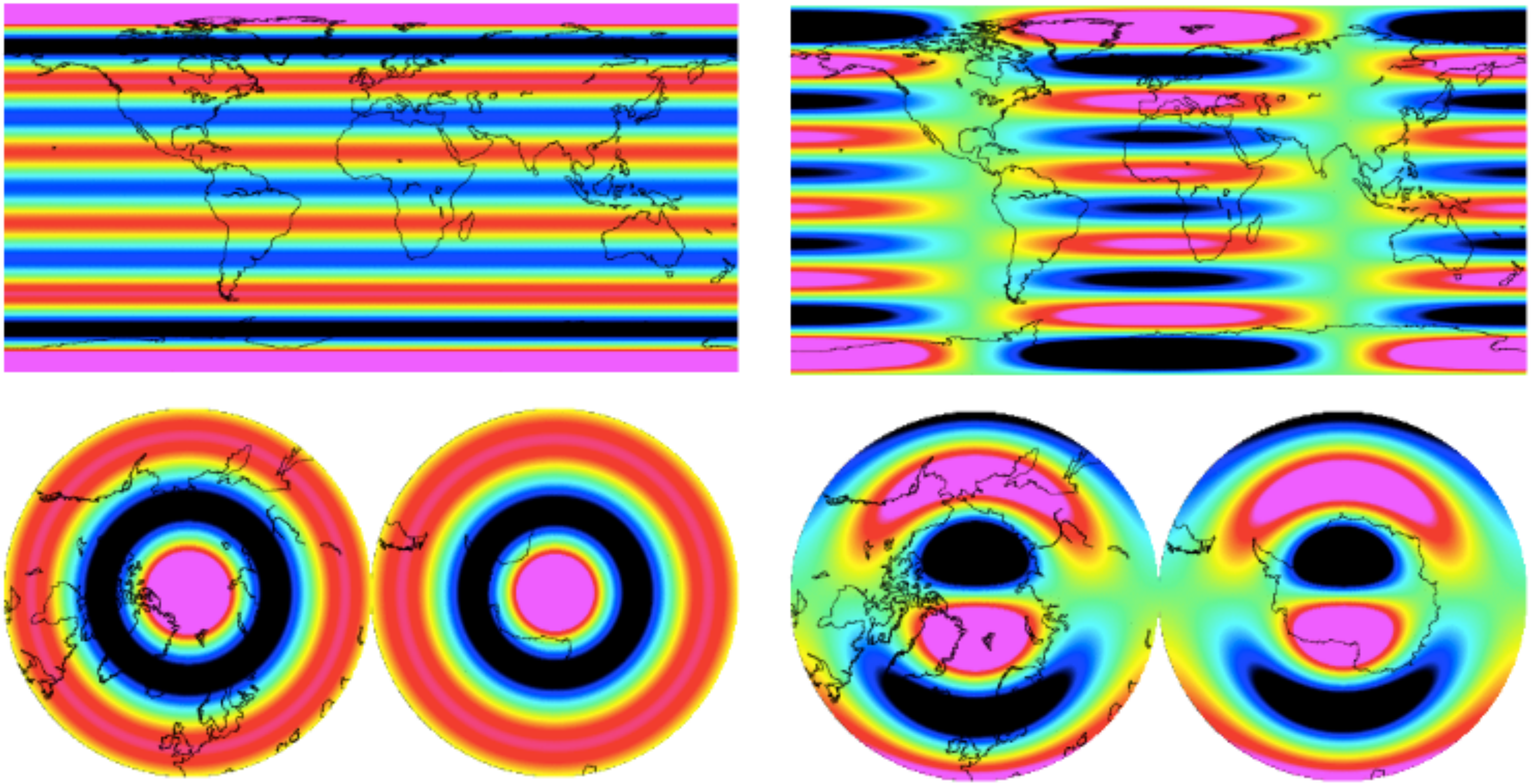
Inversion process

❖ Canonical basis: C21, C22



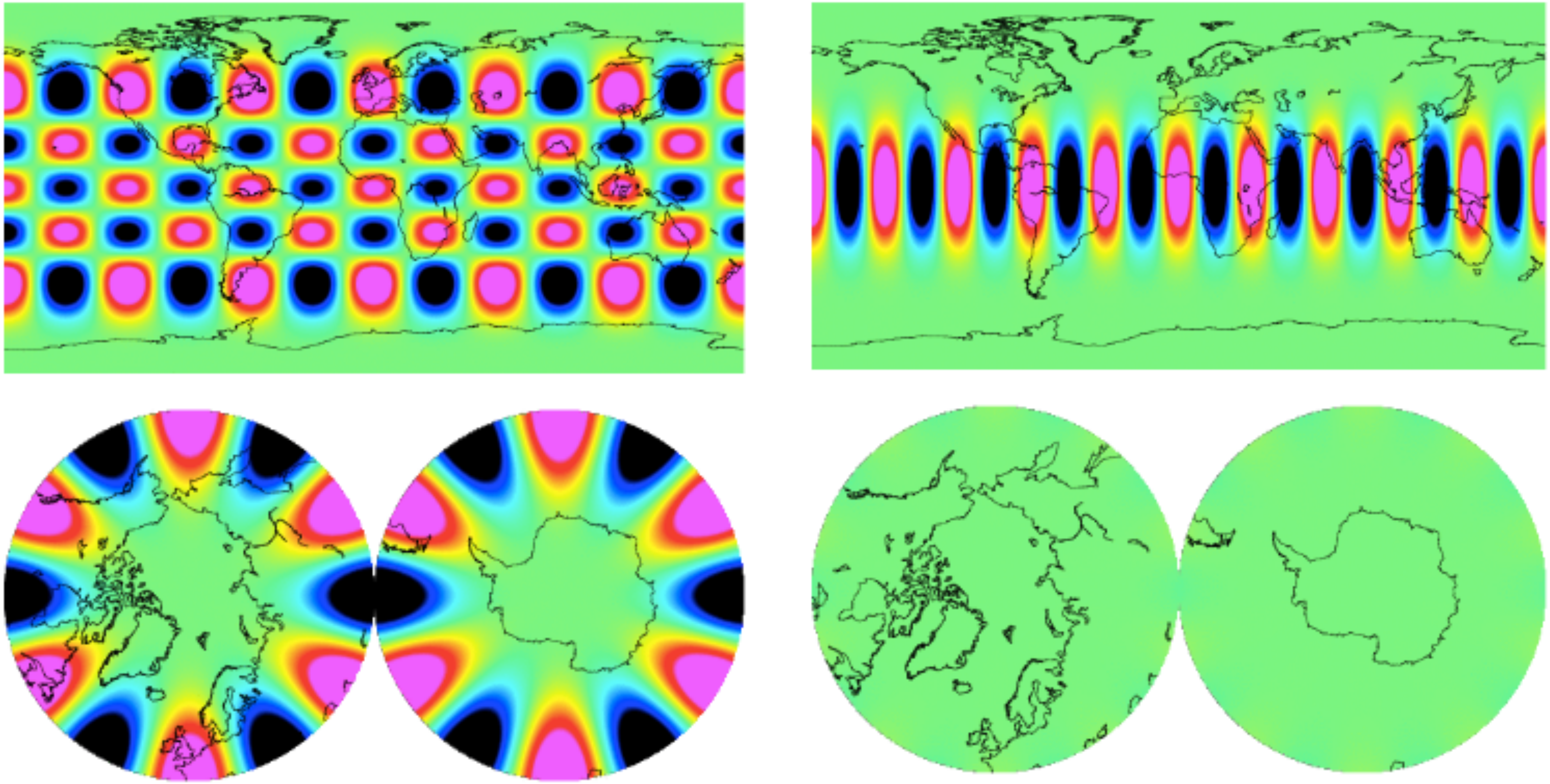
Inversion process

❖ Canonical basis: C(10,0), C(10,1)



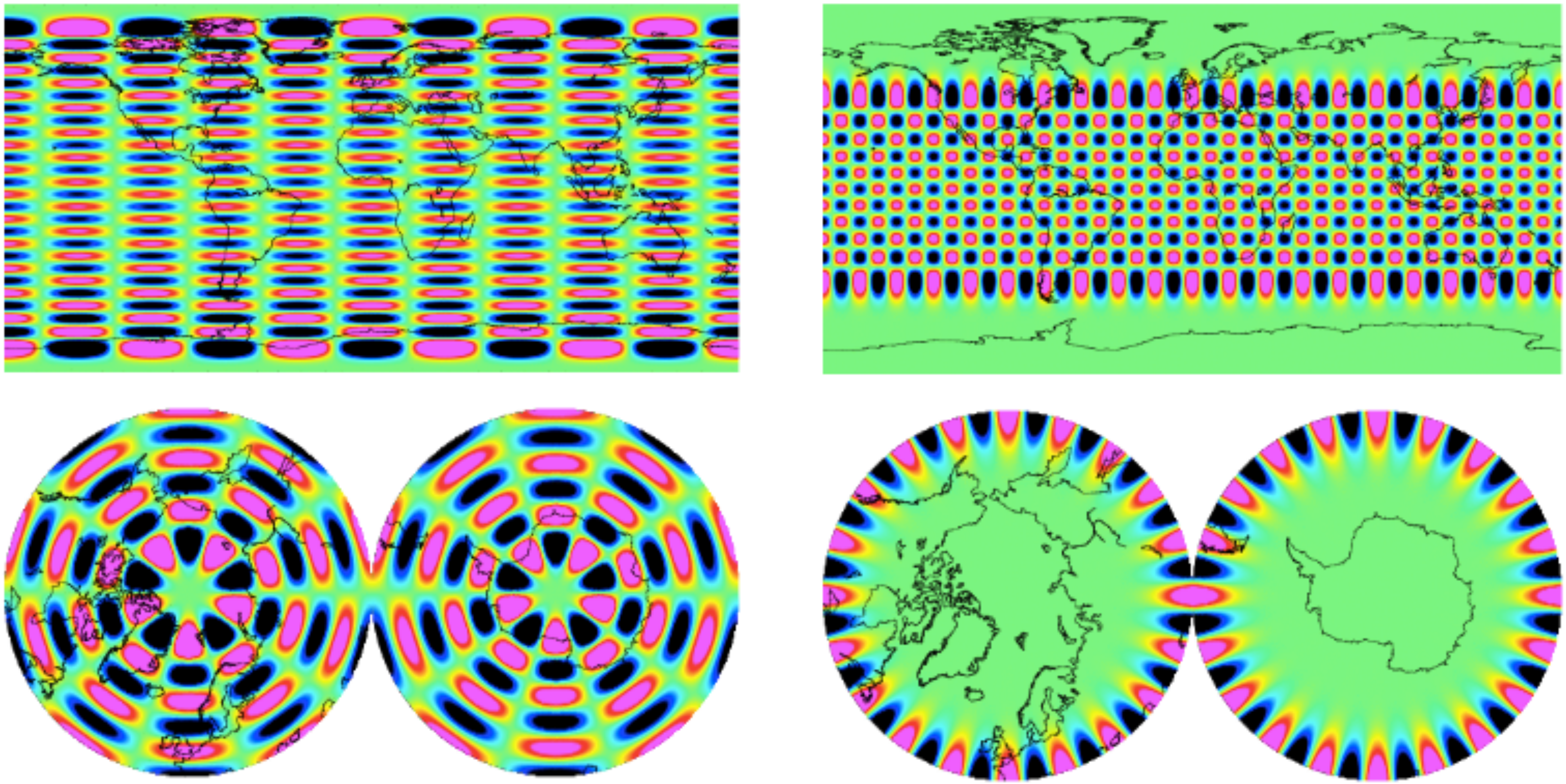
Inversion process

❖ Canonical basis: C(10,6), C(10,10)



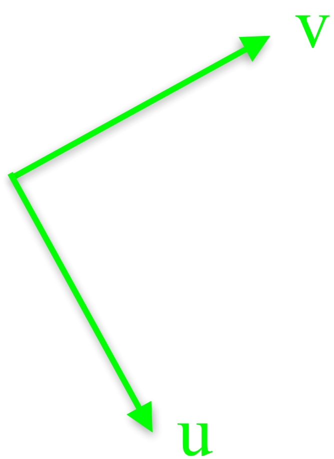
Inversion process

- Canonical basis: C(30,5), C(30,20)



Inversion process

❖ B) Basis made by the axes of the paraboloid, ranked by the « steepness of the curve »

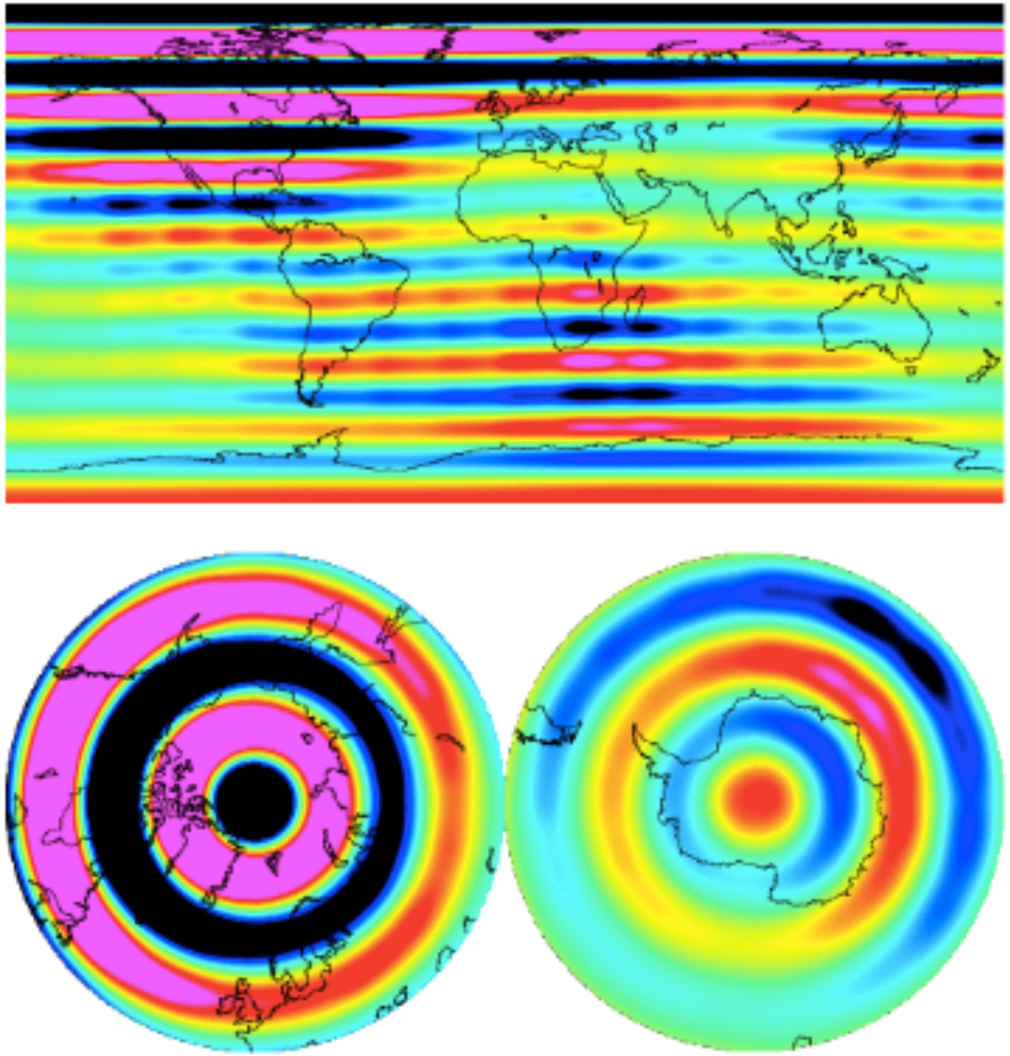


❖ Mathematically: eigenvectors of the normal matrix, ranked by higher eigenvalues

Inversion process

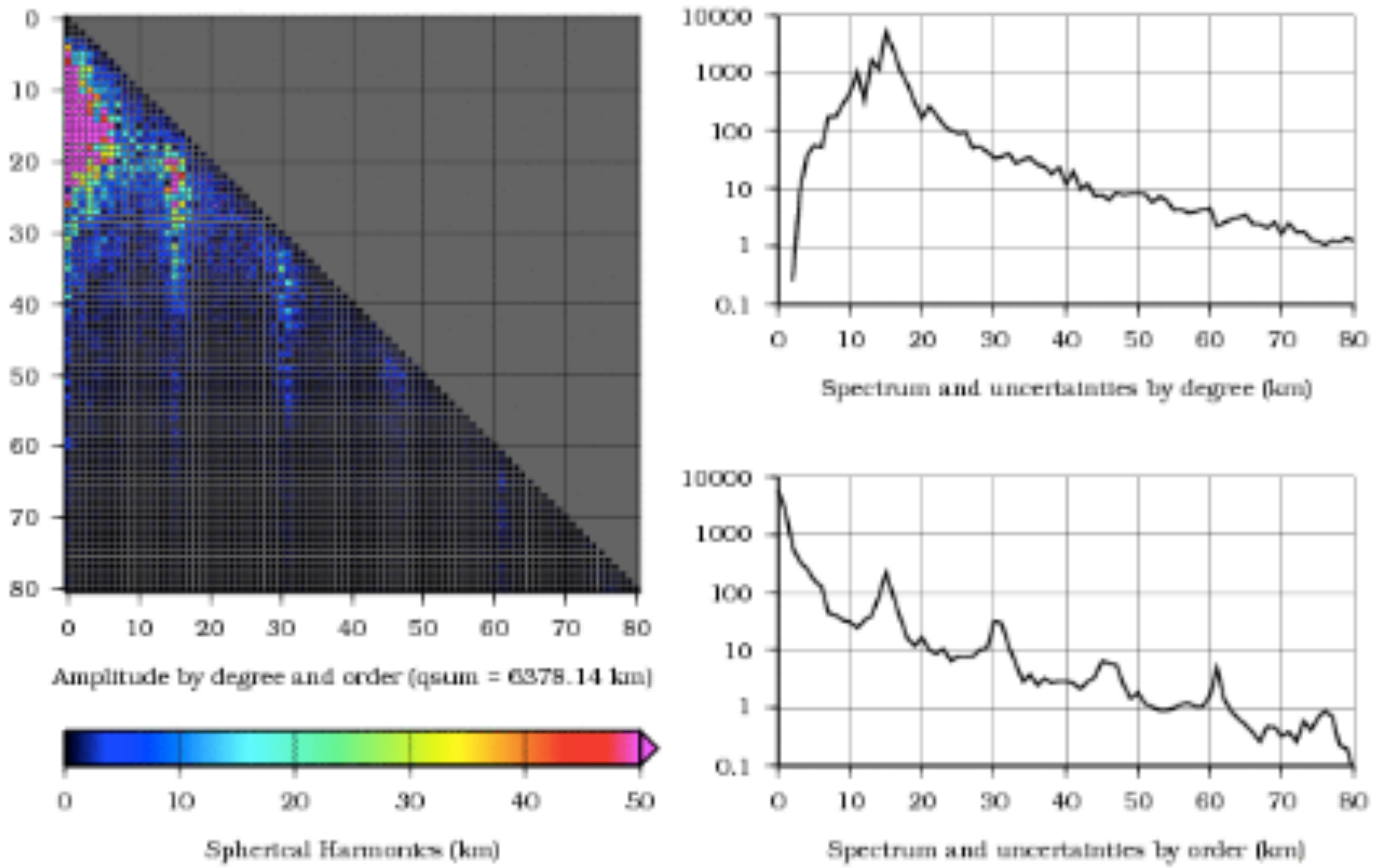
❖ Example for one monthly normal matrix

Rank no. 1



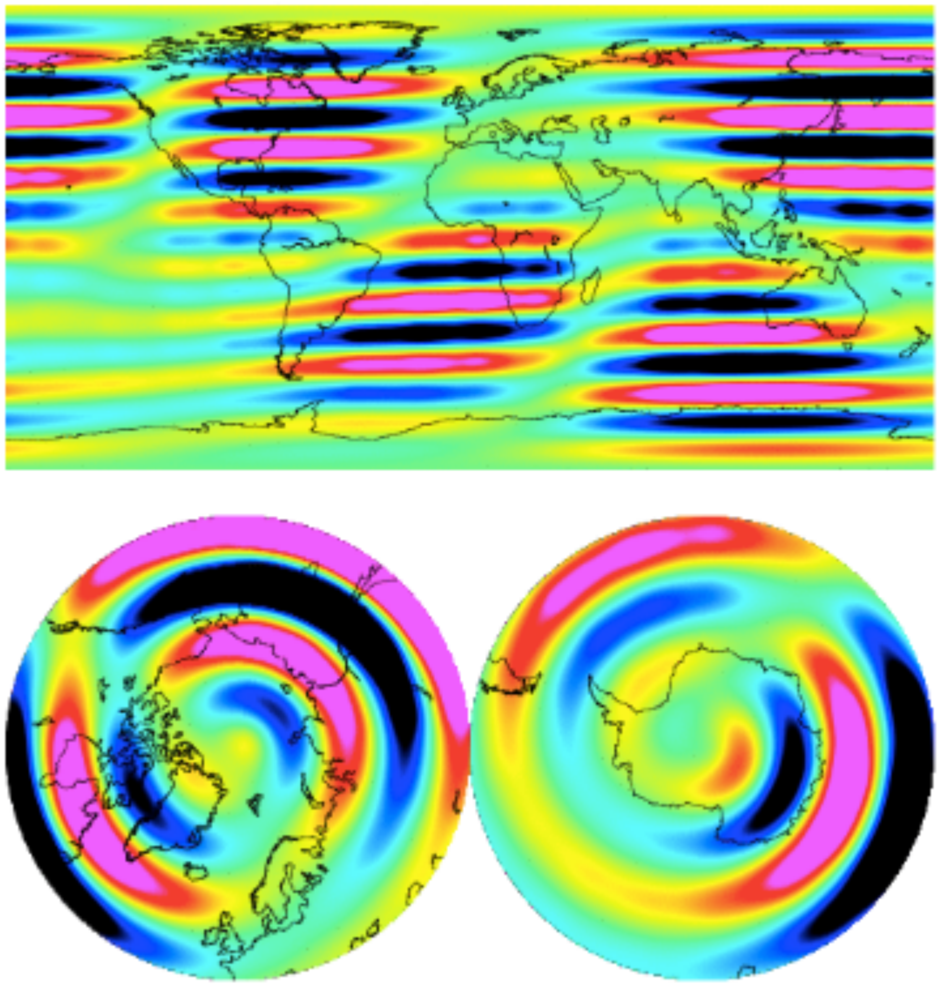
Inversion process

❖ Combination of Clm/Slm

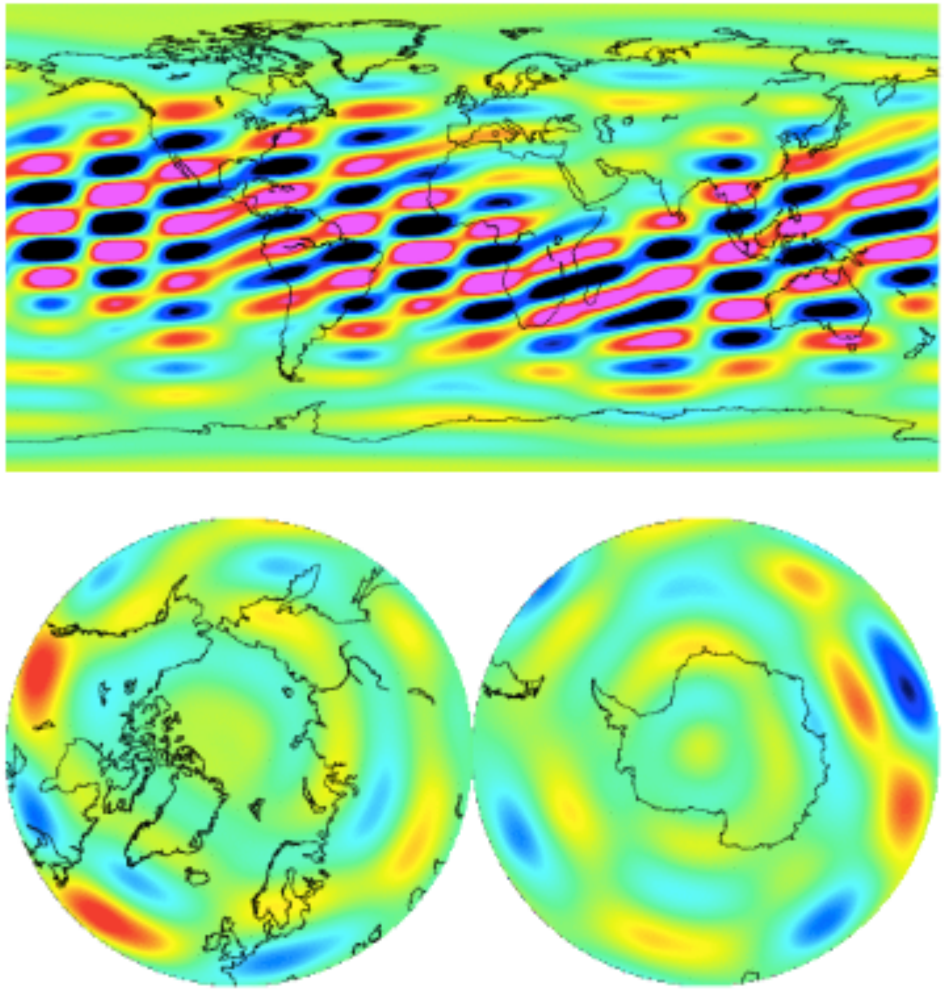


Inversion process

❖ Rank no. 10

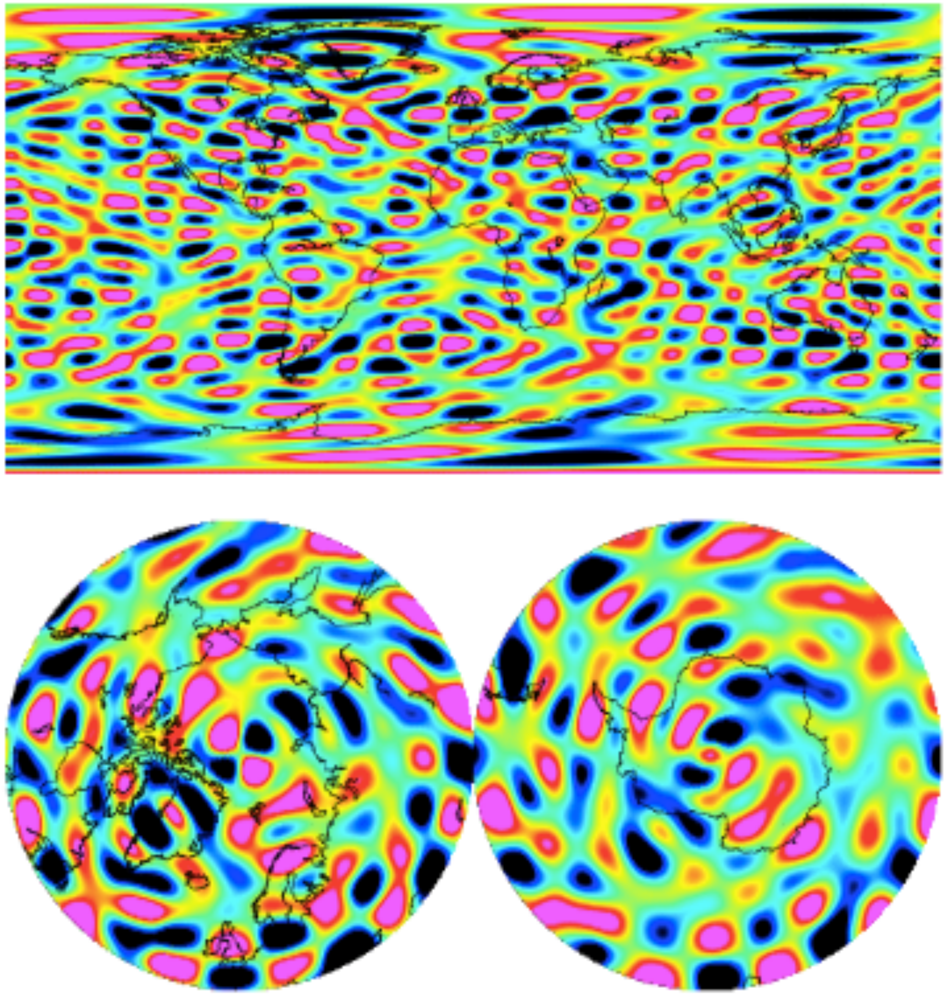


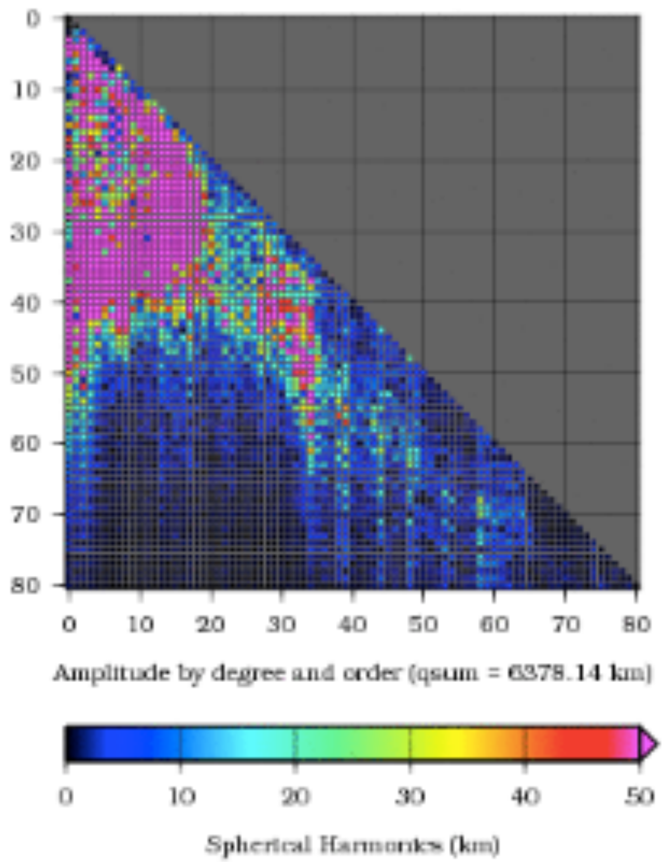
Rank no. 100



Inversion process

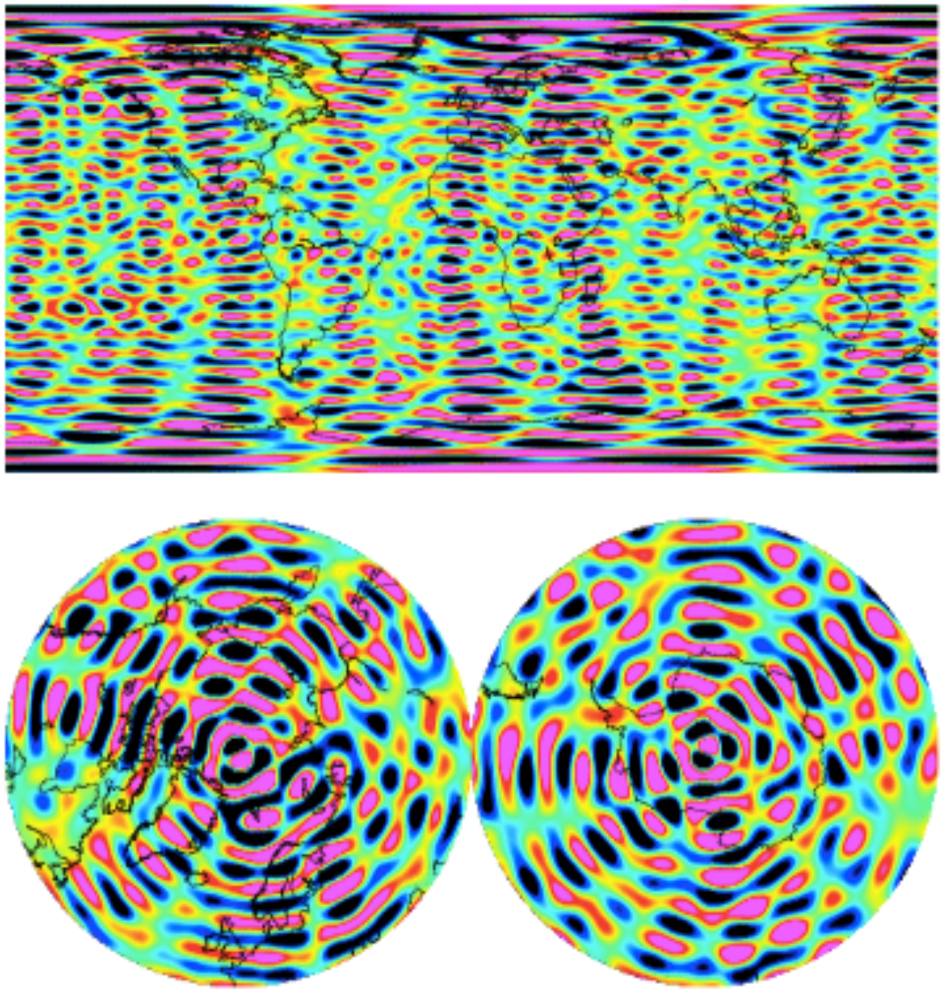
❖ Rank no. 600



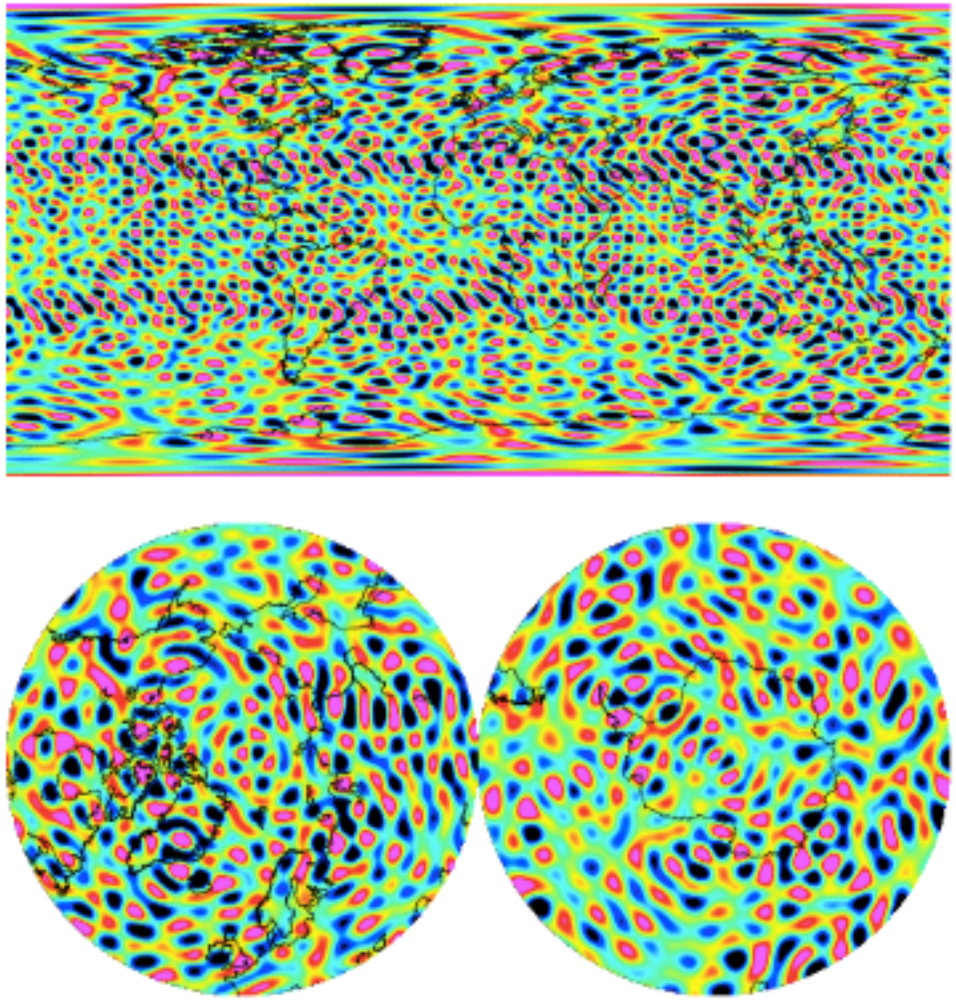


Inversion process

❖ Rank no.1500



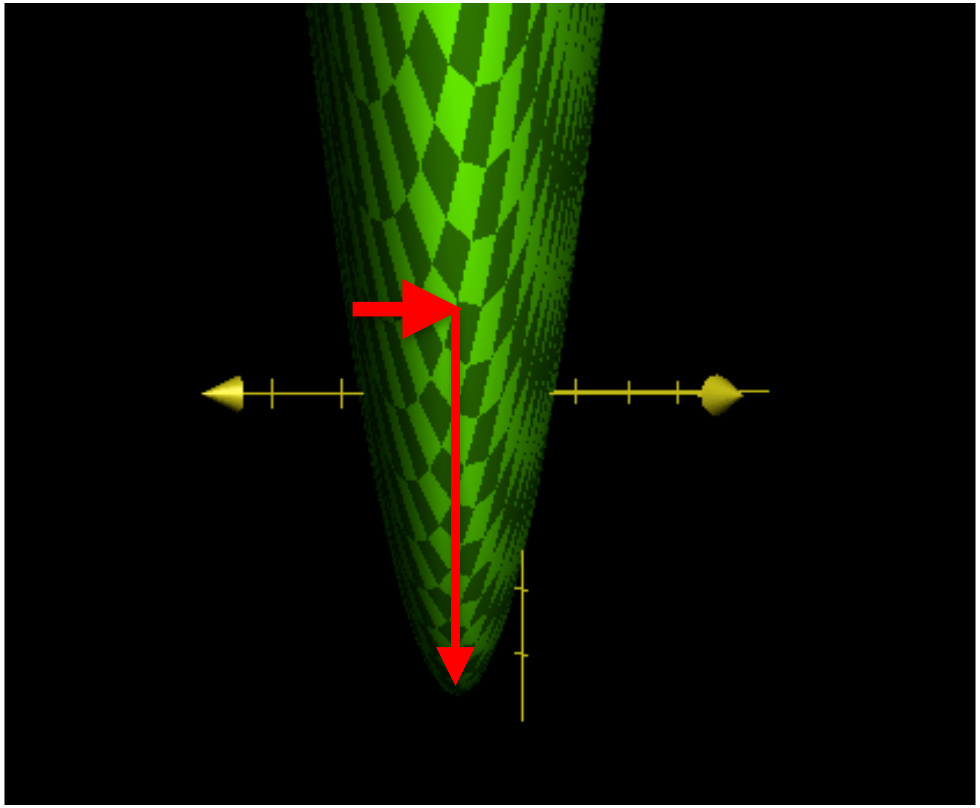
Rank no. 3000

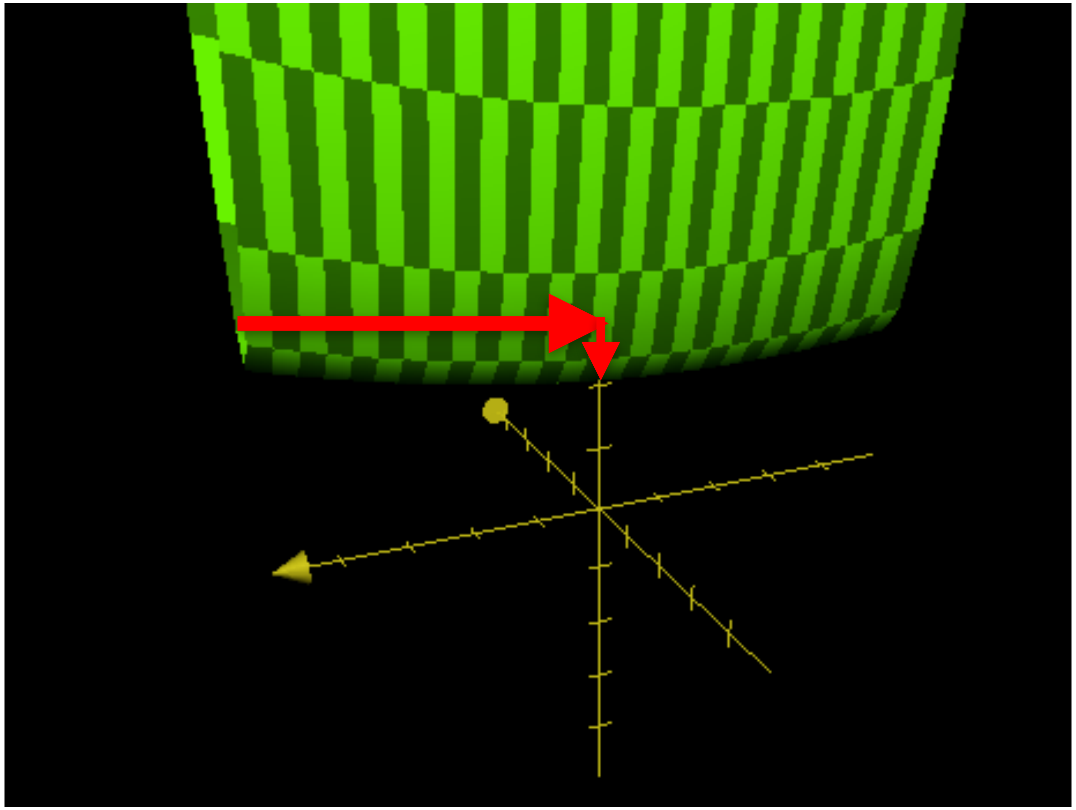


Inversion process

- **Remember:**

First ranks	**Last ranks**
Steep curve	Flat curve
Little correction	Big correction
Big improvement	Little improvement



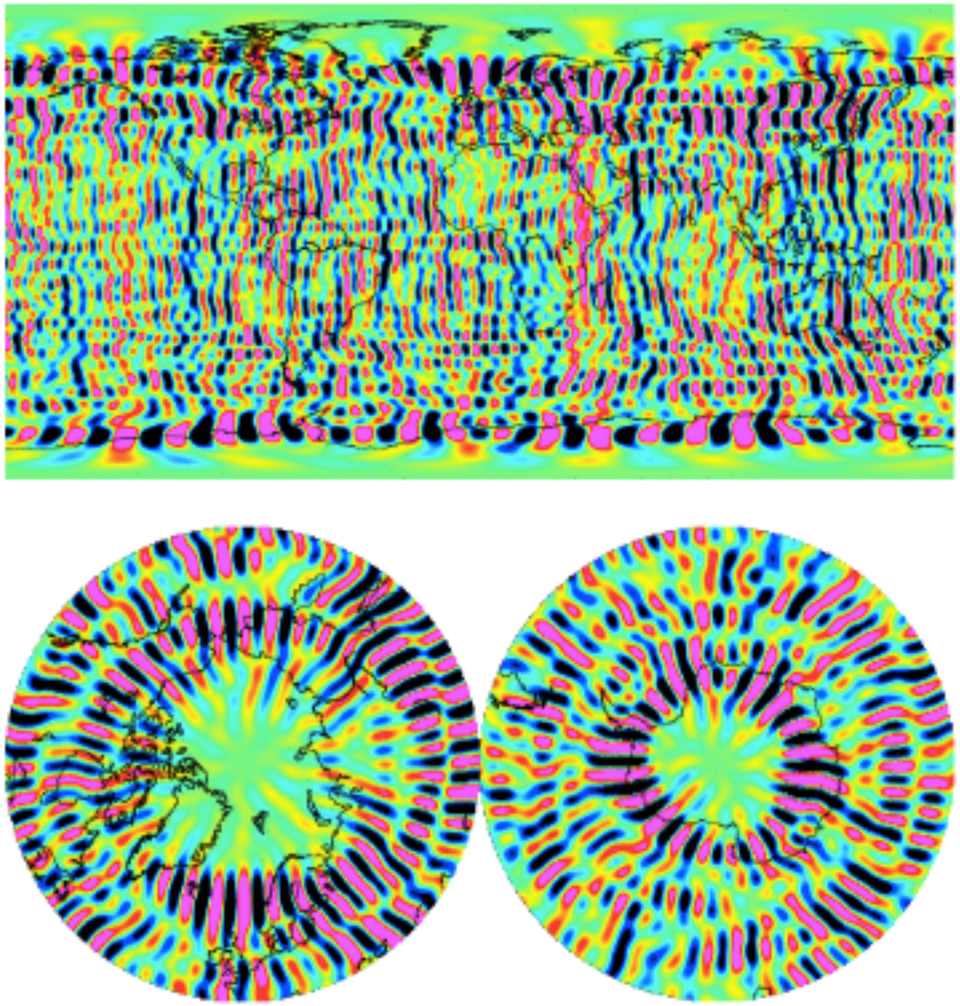


Inversion process

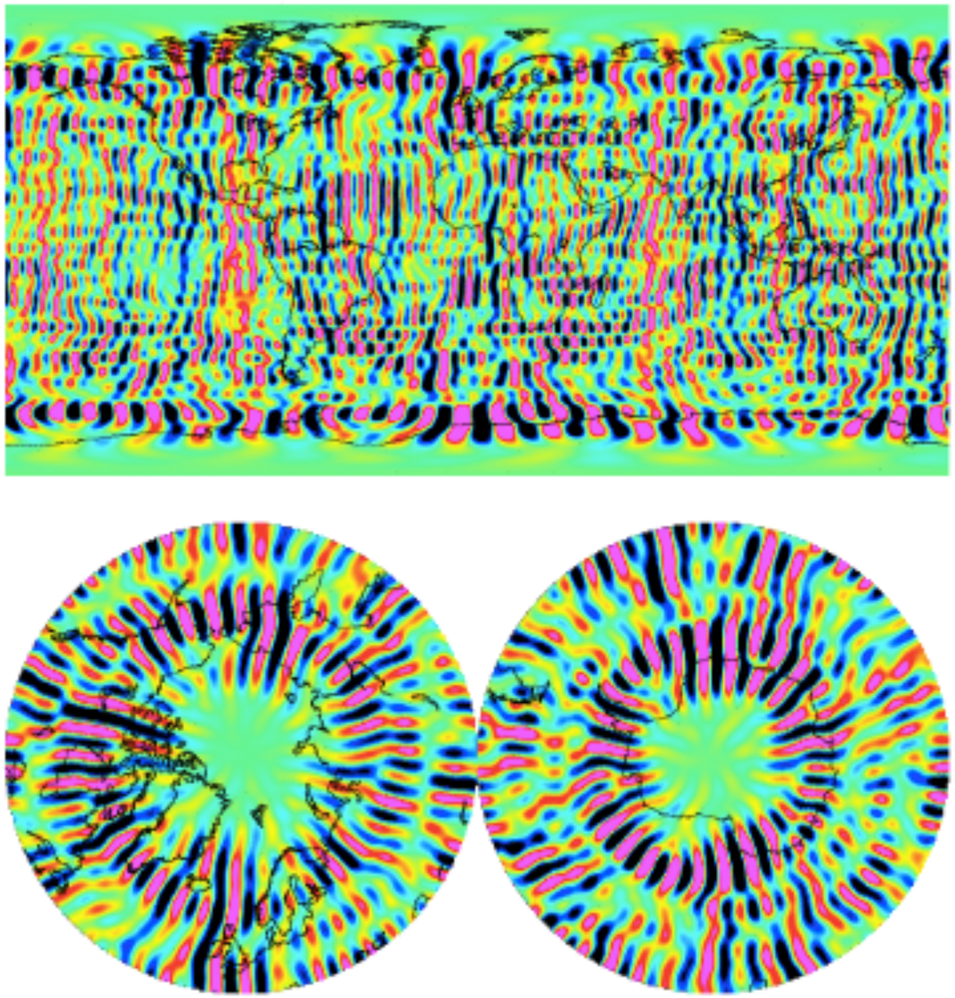
- Question: how do the last ranks look like?

Inversion process

❖ Rank no. 5900

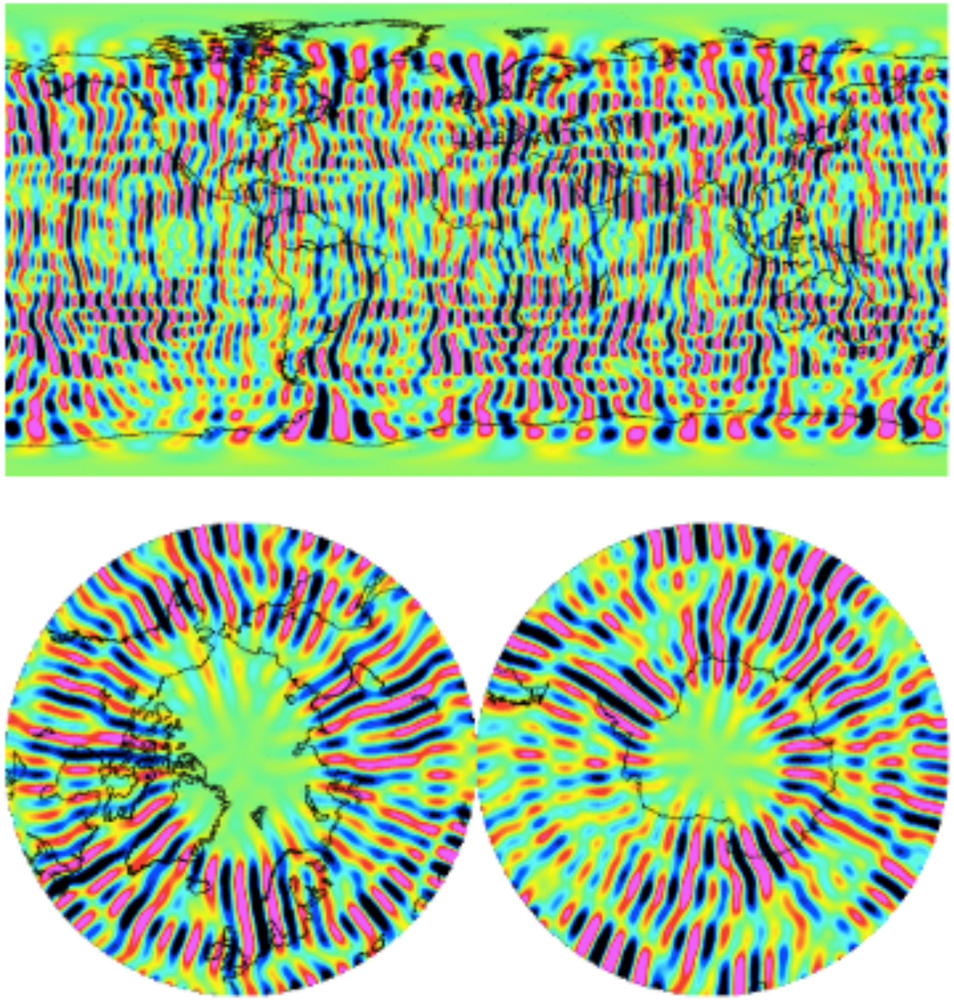


Rank no. 6000

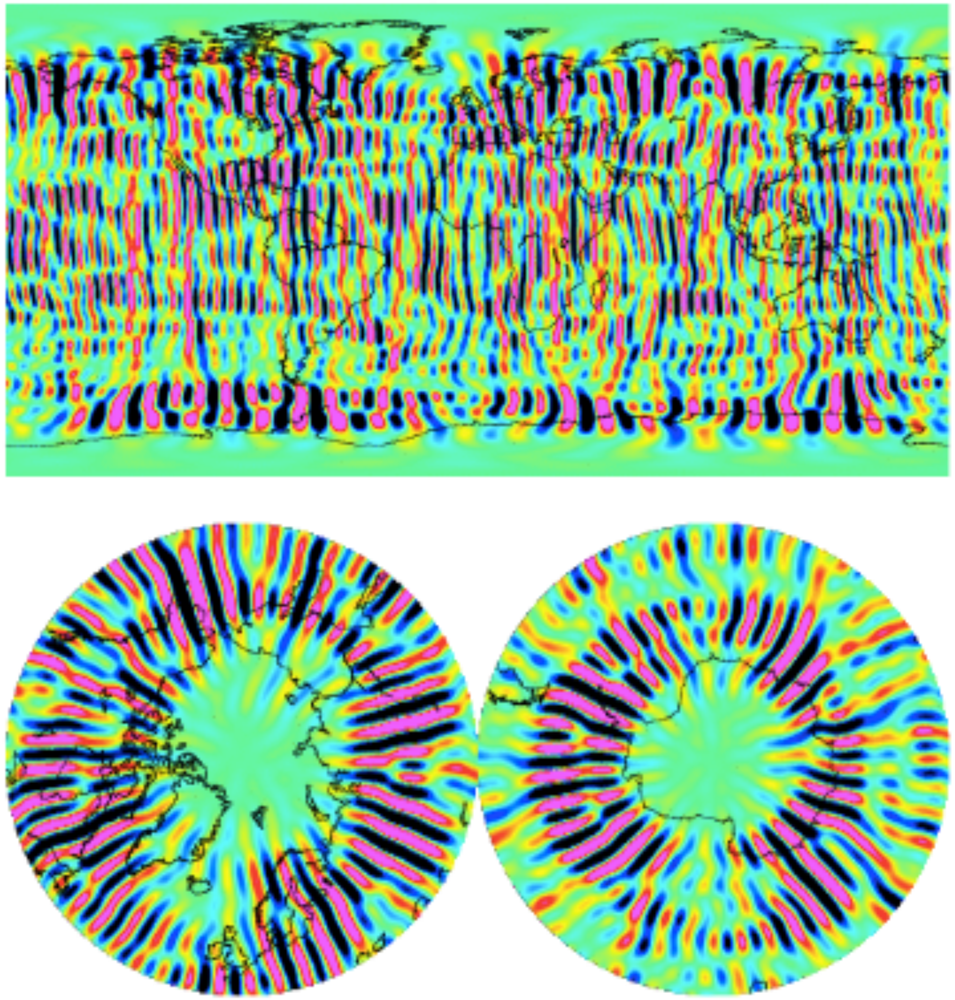


Inversion process

❖ Rank no. 6100



Rank no. 6200

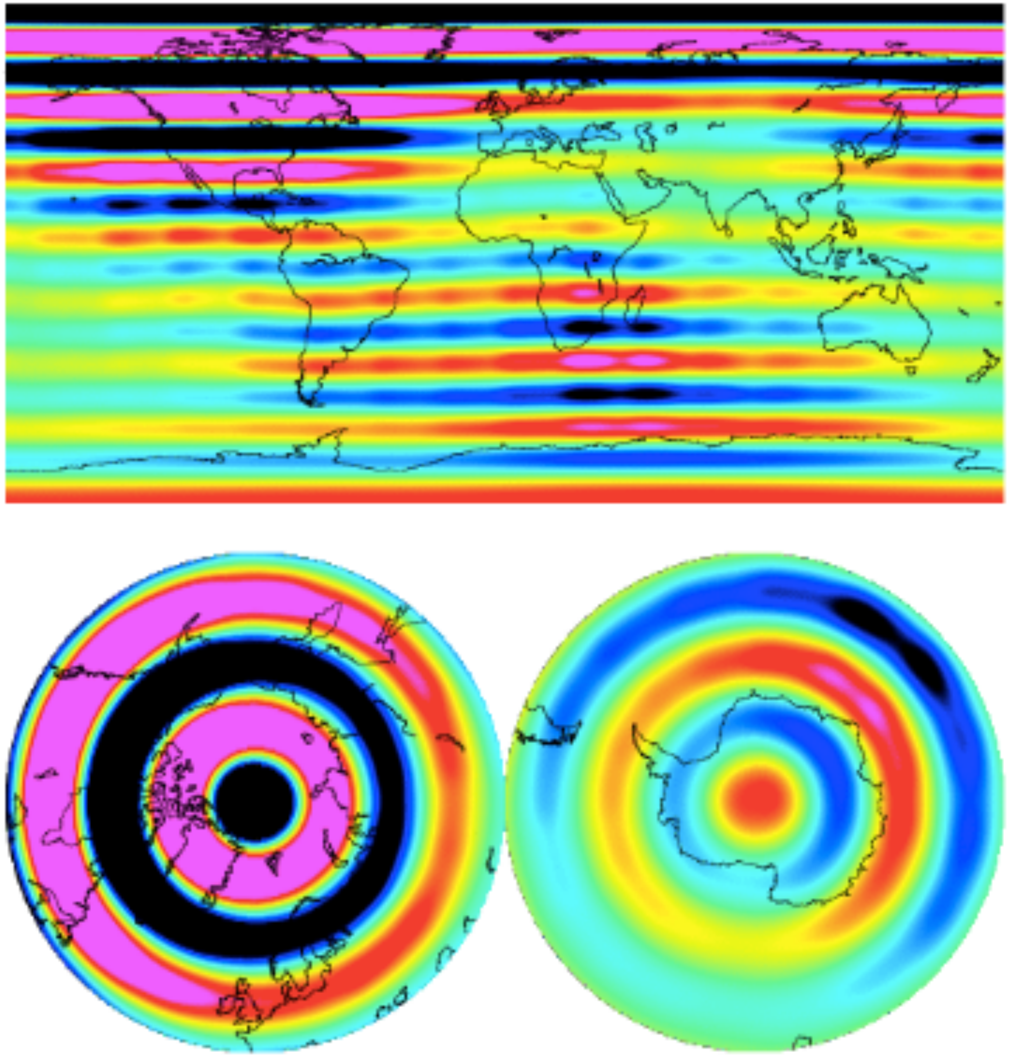


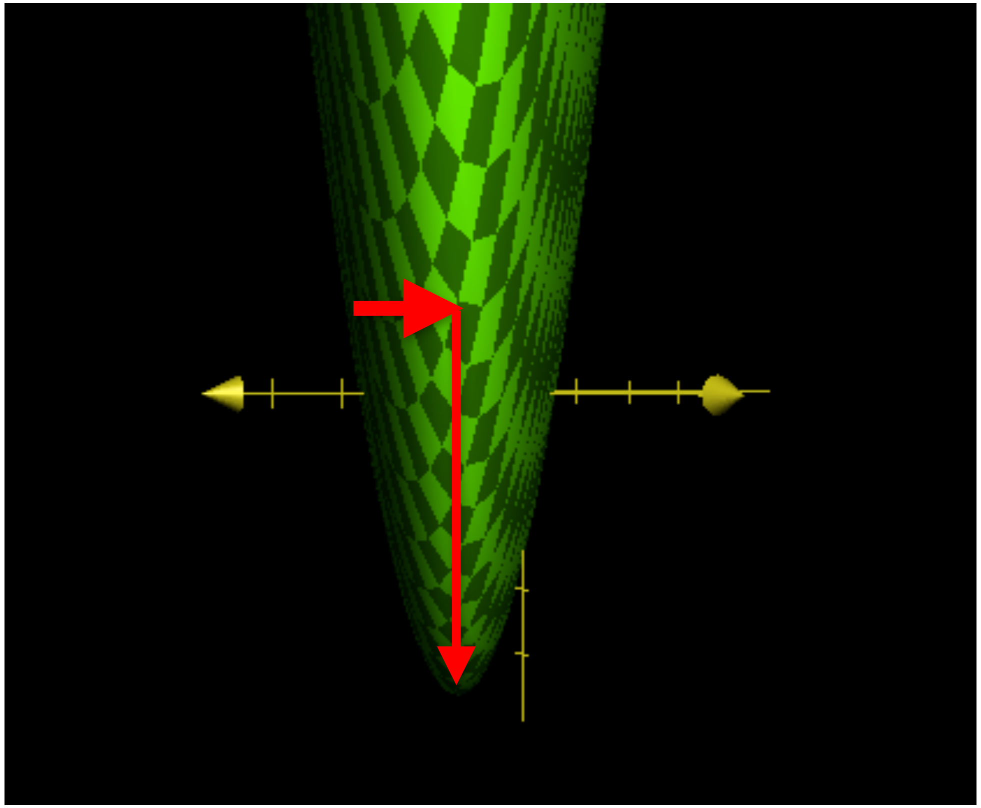
Inversion process

- The last eigenvectors correspond **exactly to the vertical stripes everybody wants to avoid**
- Yet, you have to add **a lot of it** to reach your goal (minimum)

Inversion process

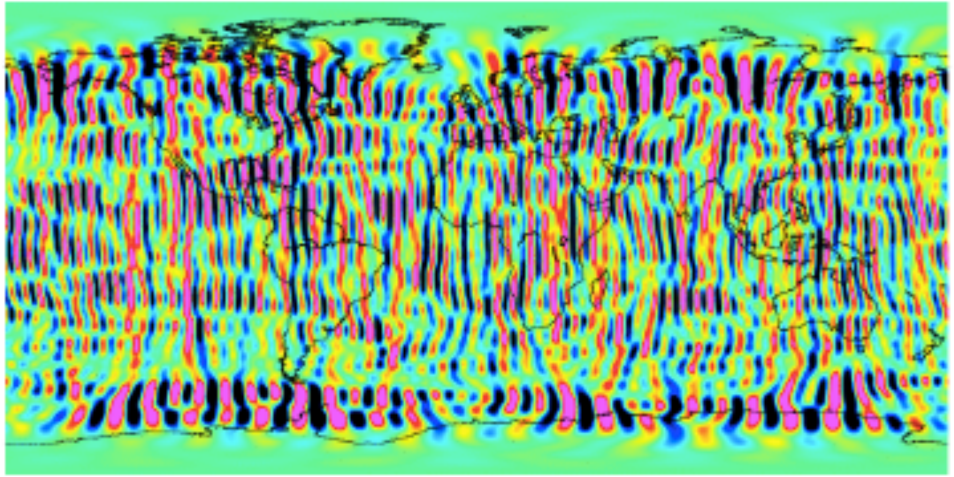
❖ Add a little bit of this…

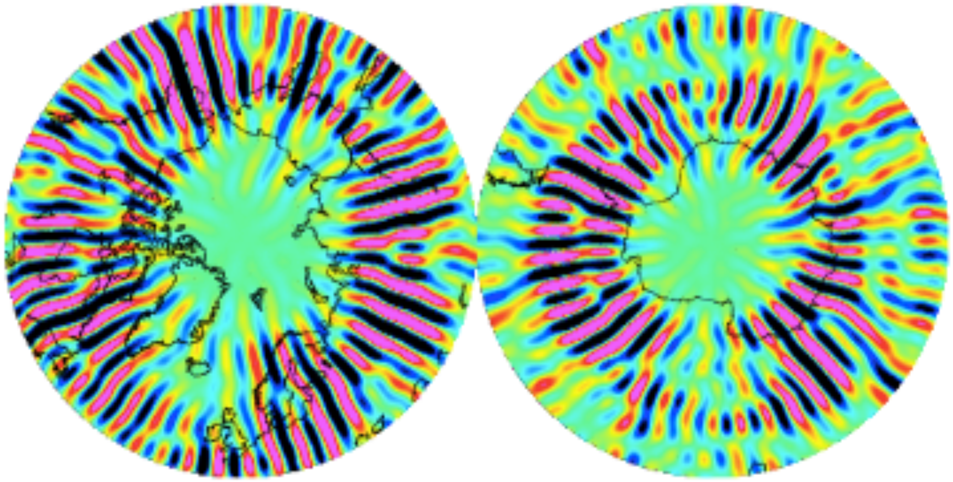


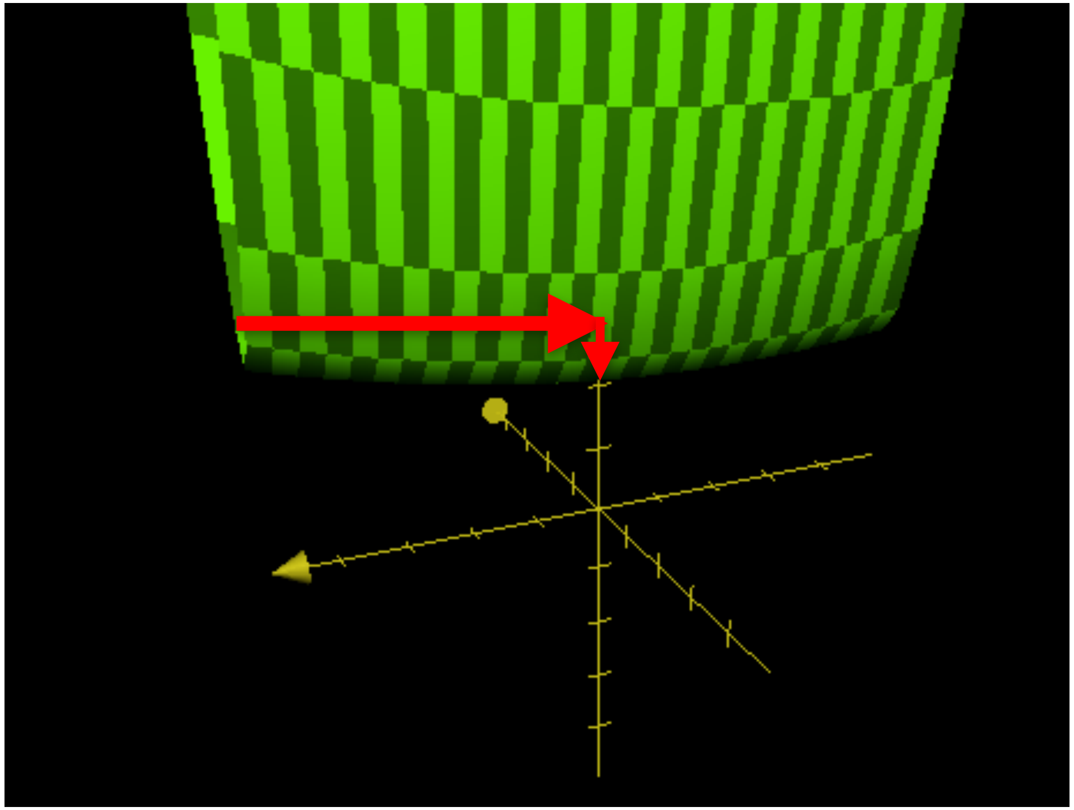


Inversion process

❖ And a lot of this…







Inversion process

❖ In other words…

❖ **The worse it is, the more you add.**

❖ Does it make any sense at all?

Inversion process

- Along flat directions, there is no good reason to search for the minimum

- It introduces stripes with no improvement on the residuals.

- Suggested solution: don't introduce the stripes

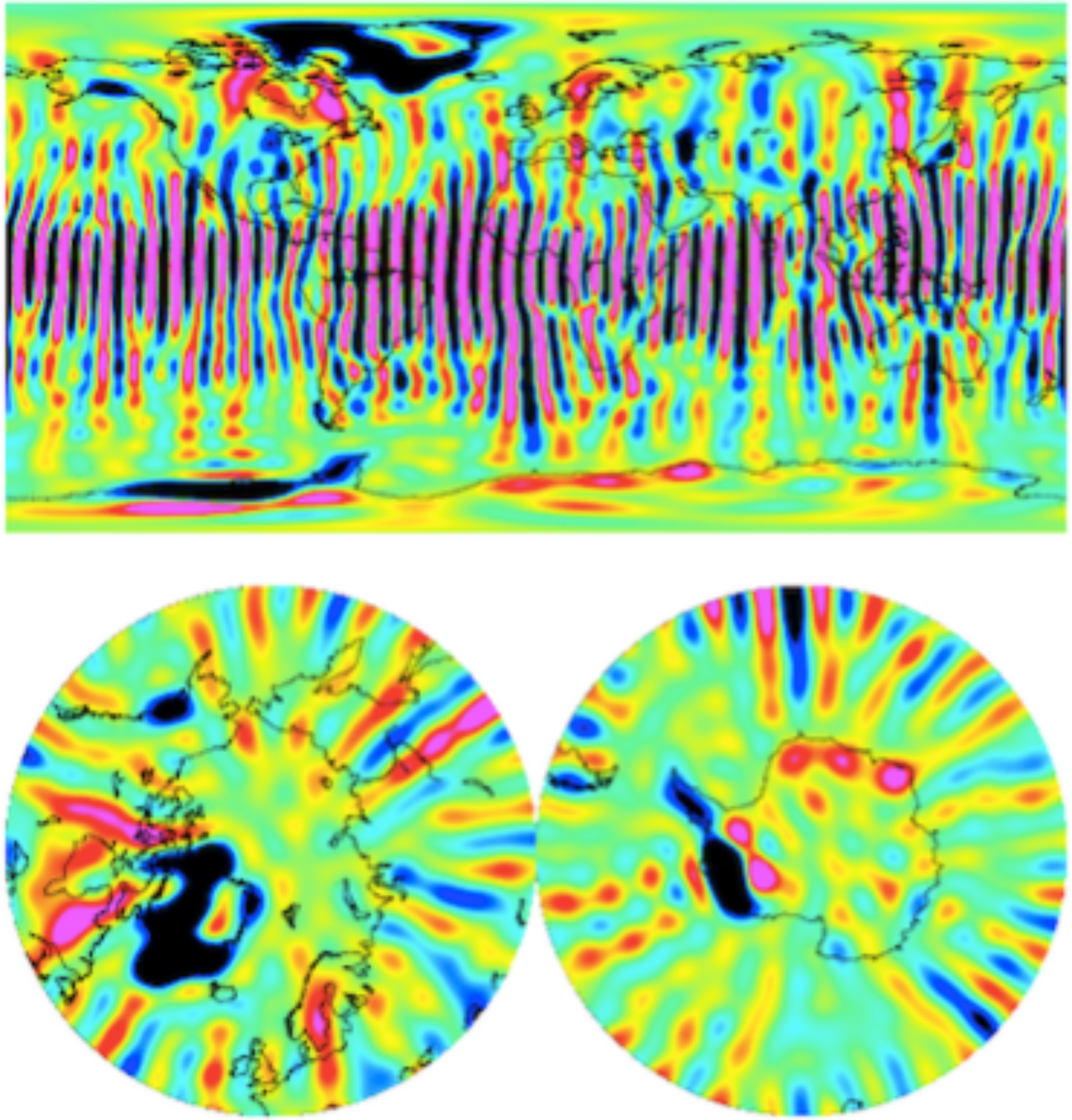
Inversion process

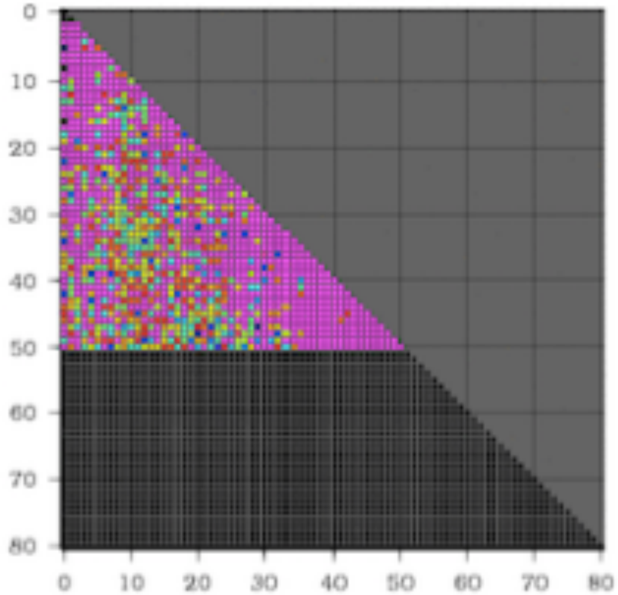
- A quick illustration of the results
  - One normal matrix up to degree 80 (6400 coefficients)

- Inversion of 2500 parameters:
  - Case 1) All SH coefficients up to degree 50
  - Case 2) First 2500 eigenvectors

Inversion process

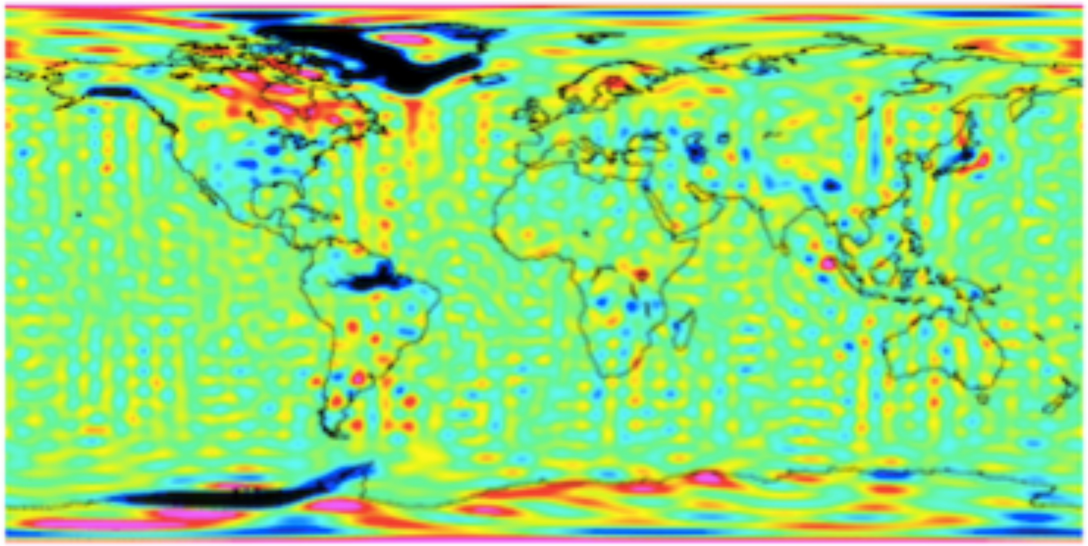
❖ Case 1: all coefficients up to 50 (2500 parameters)

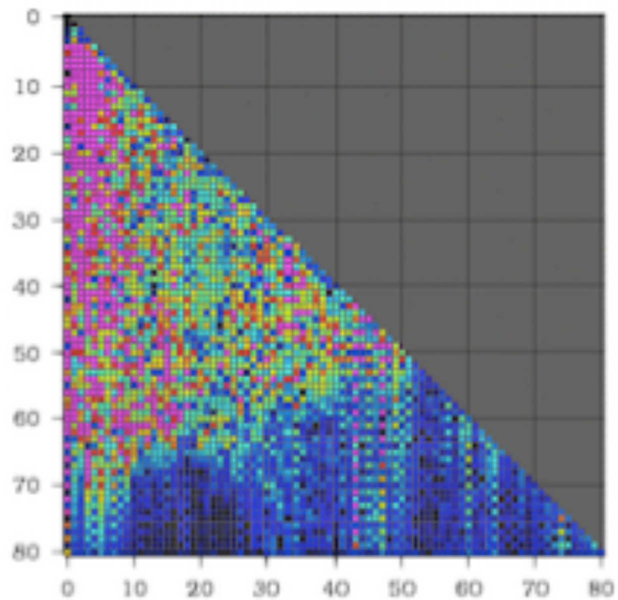


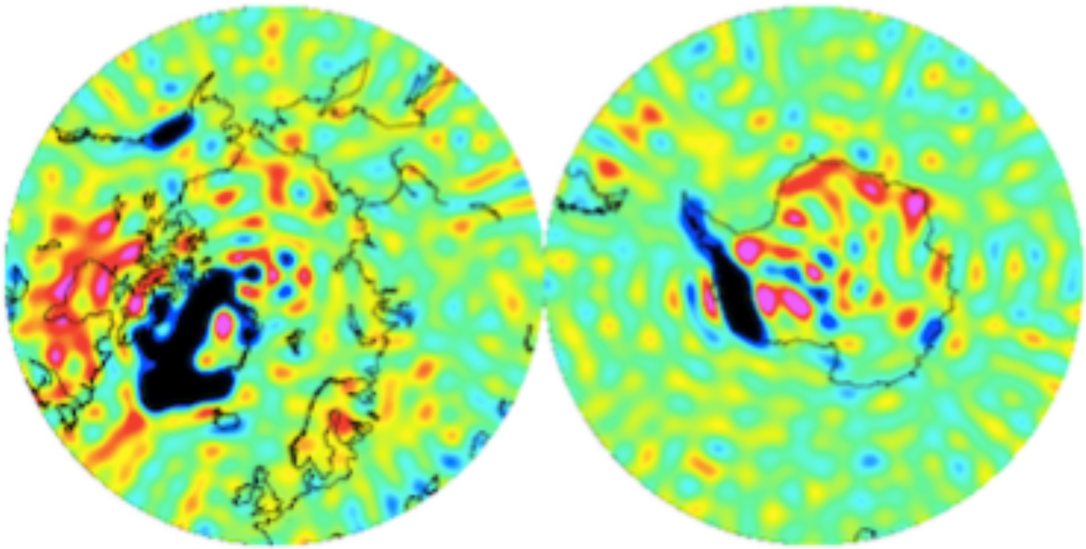


Inversion process

❖ Case 2: first 2500 eigenvectors







❖ Same number of parameters

❖ No filtering

❖ No stripes

Inversion process

- Conclusion: we avoid most of the stripes and therefore our solutions don't require filtering

- Although our method looks « different » from other groups, it is worth trying: grgs.obs-mip.fr/grace

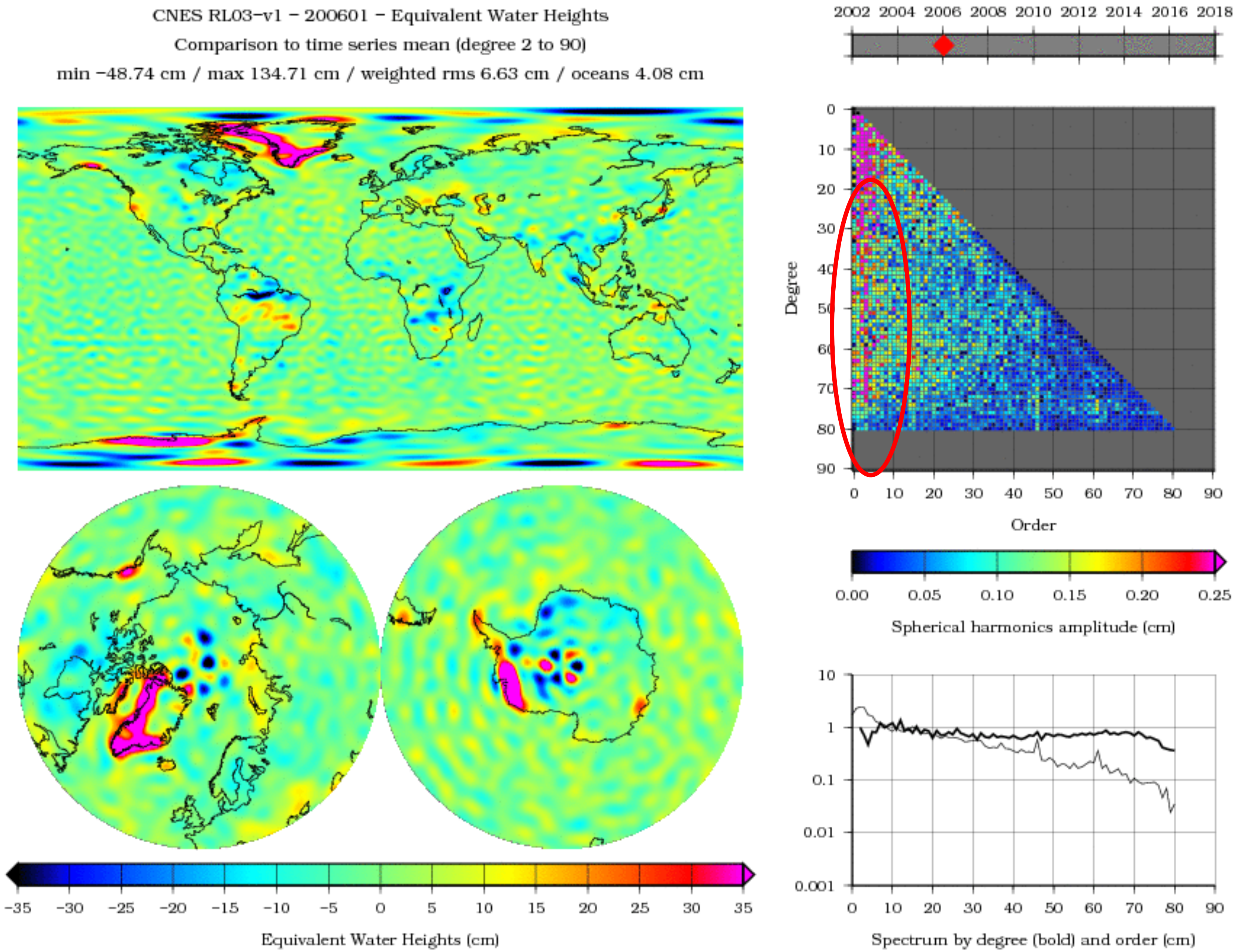
- Compare our solutions with: DDK5 or DDK6 filtered solutions, or with non-GRACE data.

Inversion process

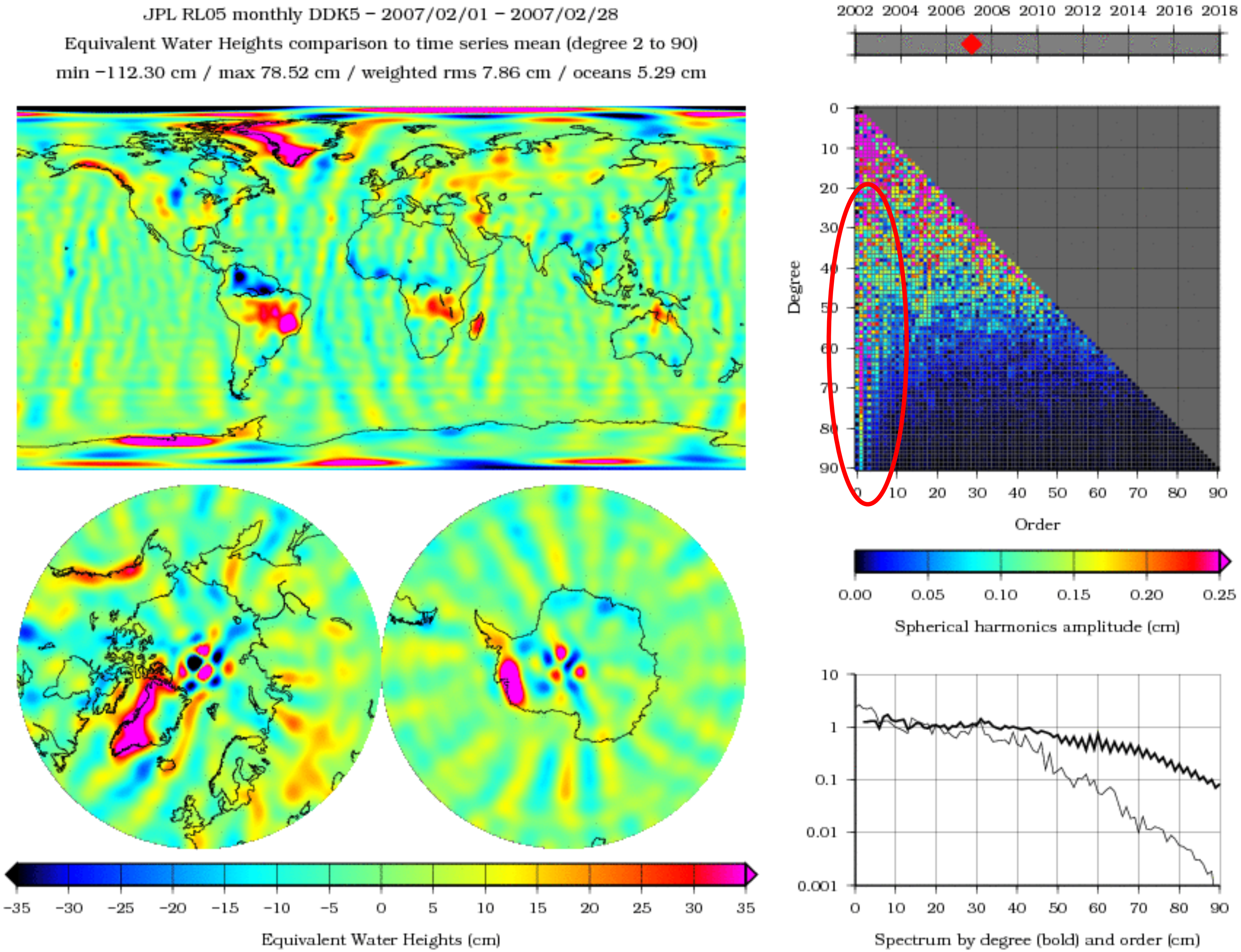
- More detail:
  - It looks simple, but the method is <u>not</u> as straightforward as suggested.

- Typical issues:
  - Choice of the truncation level
  - Because of the truncation, some low-degree coefficients are not solved…
  - … which produces undesirable side effects, such as noise at the poles (example: CNES RL03-v1).

Inversion process



Inversion process



Inversion process

- Solution: we need to do a **two-step process**:

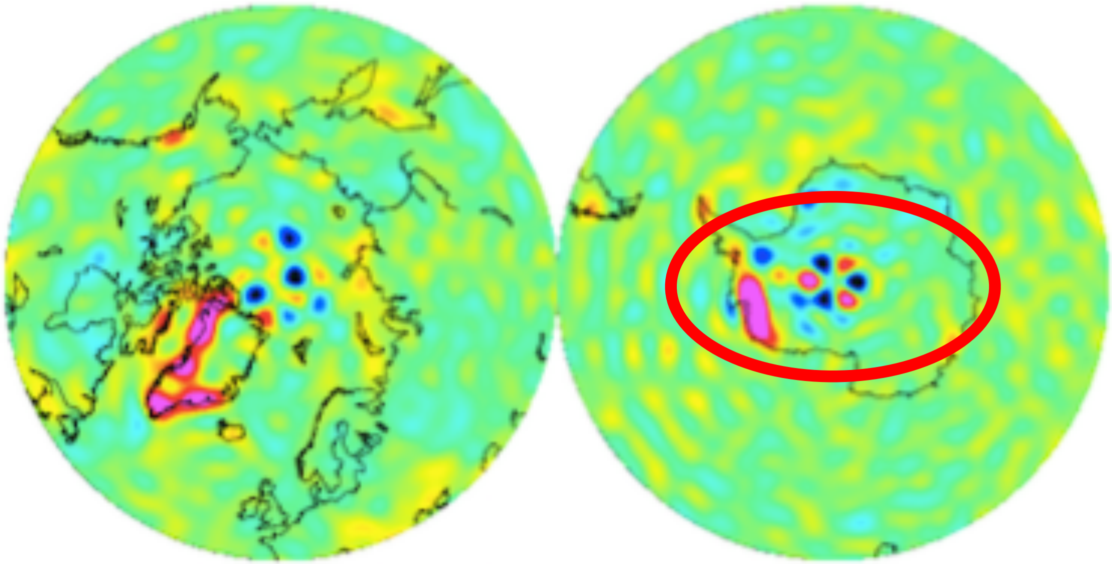
- 1) Standard inversion of the low degrees, in order to avoid bad initial values with side effects on other coefficients.

- 2) Injection of the solution as initial values of the normal equation, and then truncated SVD.

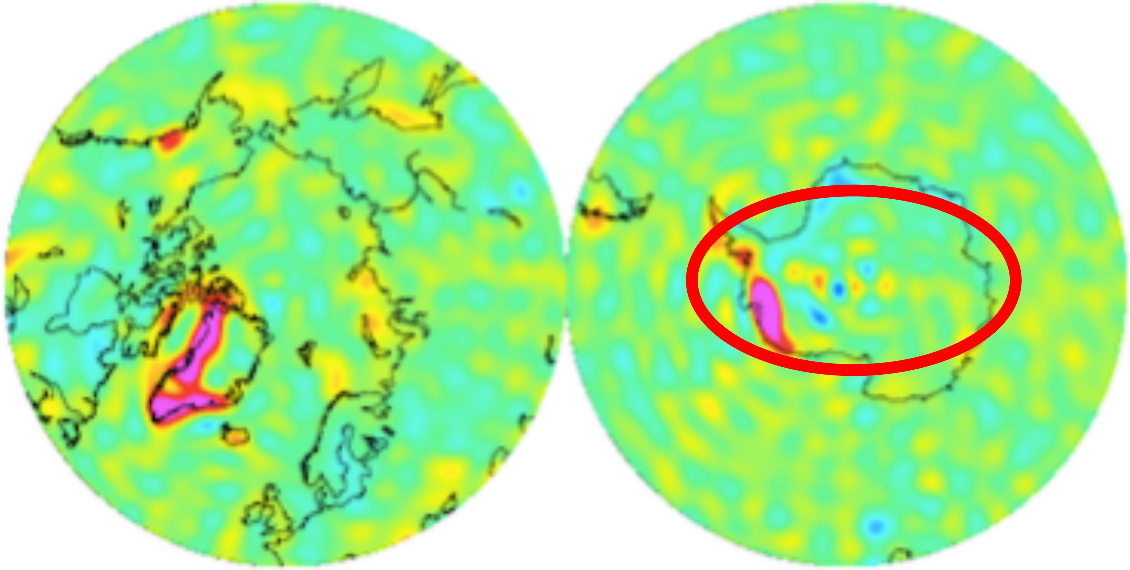
- An article being prepared to describe our process in detail.

Inversion process

- CNES RL03-v1
  2006/01
  One-step
  inversion



- CNES RL03-v2
  2006/01
  Two-step
  inversion



Conclusions

- Current status:
  - RL03-v1: problems at the poles.
  - RL03-v2: solved the problems at the poles. Problem recently identified in our C21/S21 coefficients between 2003 and 2012.
  - RL03-v3: just released at grgs.obs-mip.fr/grace

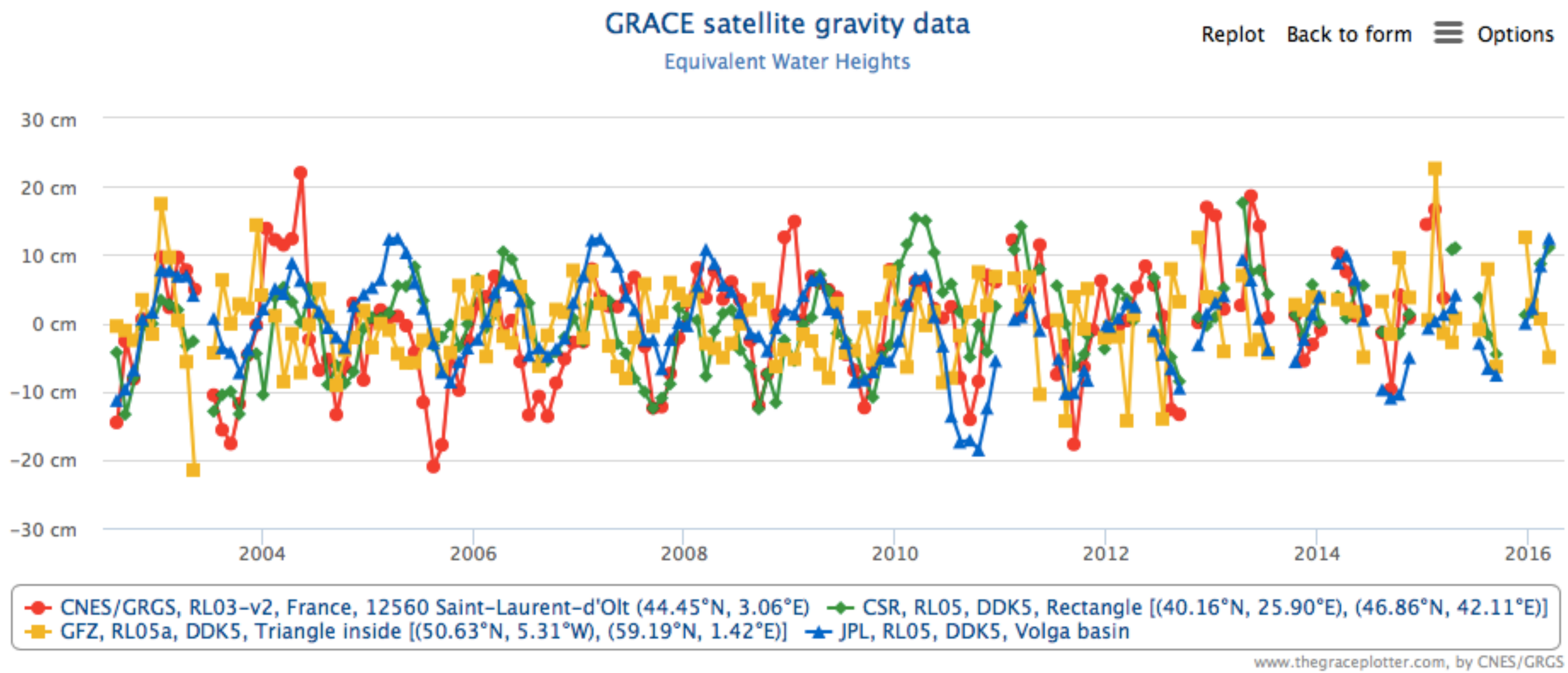
- **RL03-v3 is now available on our website** until March 2015. We will complete the series until June 2016 by the end of October.

Conclusions

- Poster session for questions and discussion
- Reminder: interactive website
  - thegraceplotter.com / plot.egsiem.eu

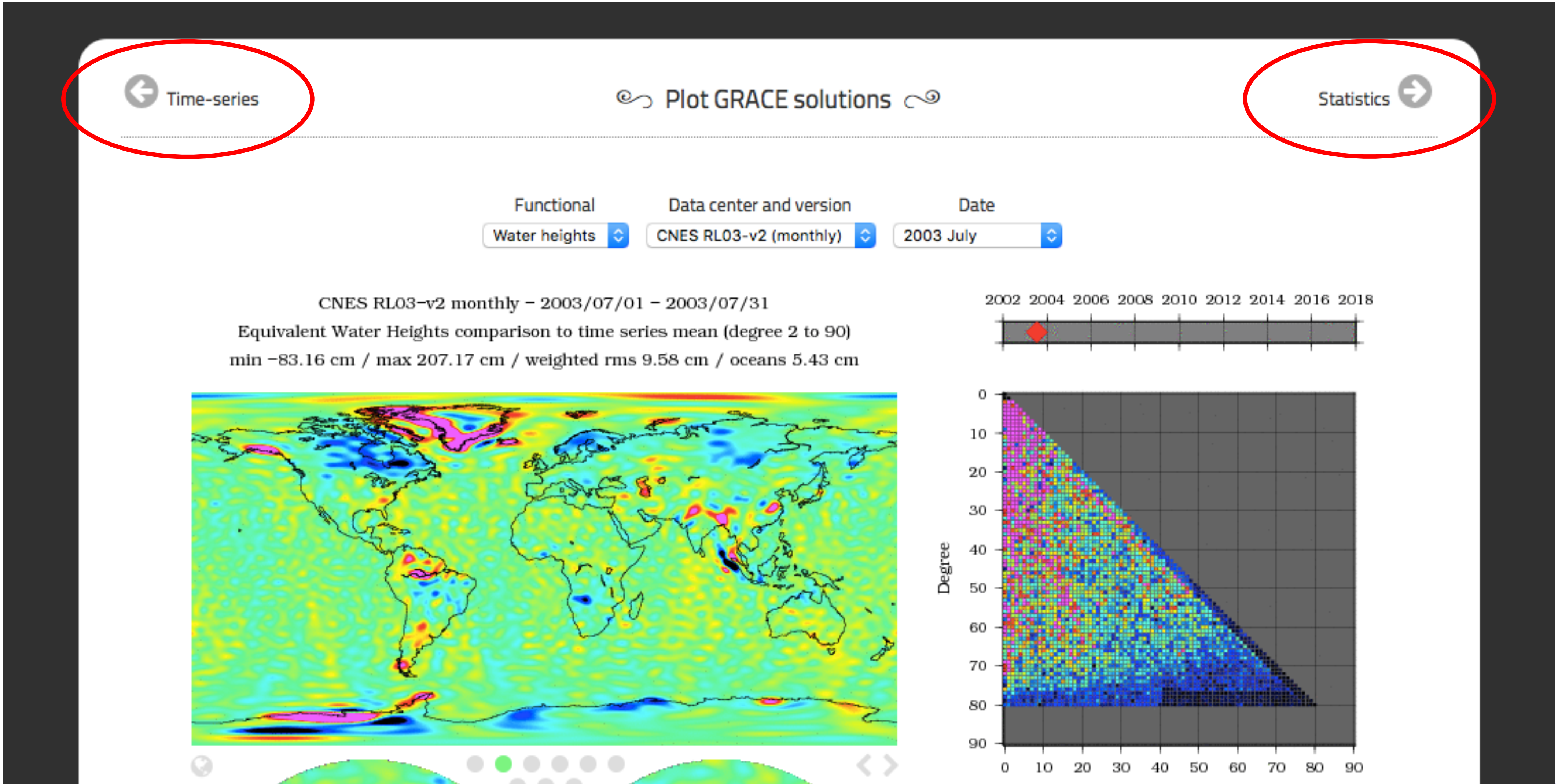
Conclusions

❖ Geographical extraction + group comparisons



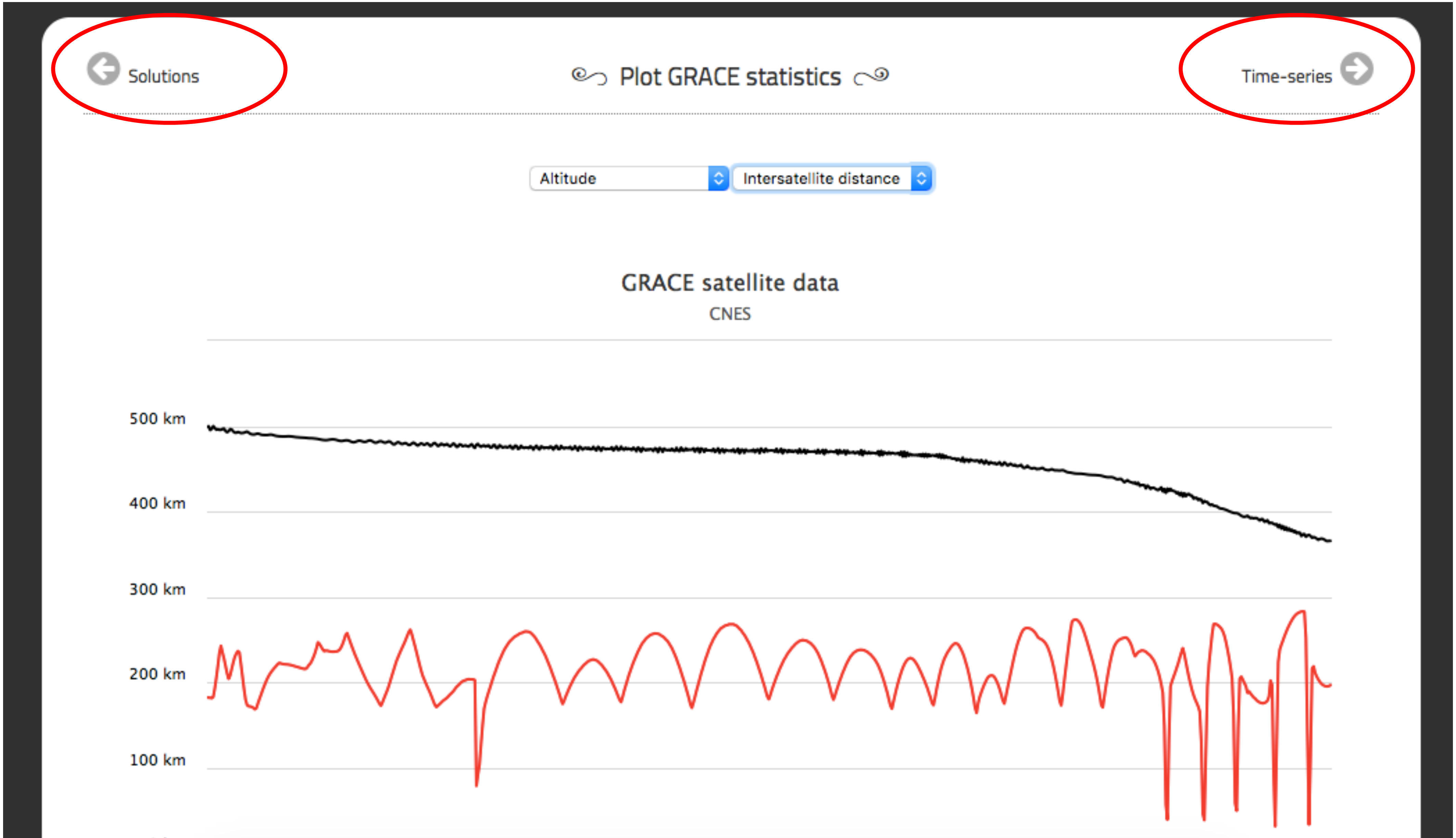
Conclusions

❖ Browse images of every group (CNES/GRGS, CSR, JPL, GFZ, AIUB, TUGRAZ, TONGJI, HUST)



Conclusions

❖ Other statistics



Thank you

❖ Thank you for your attention