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European Gravity Service for Improved Emergency Management

GRACE Gravity Field Determination using Refined Acceleration Approach: Preliminary results

ULUX—WP2 progress

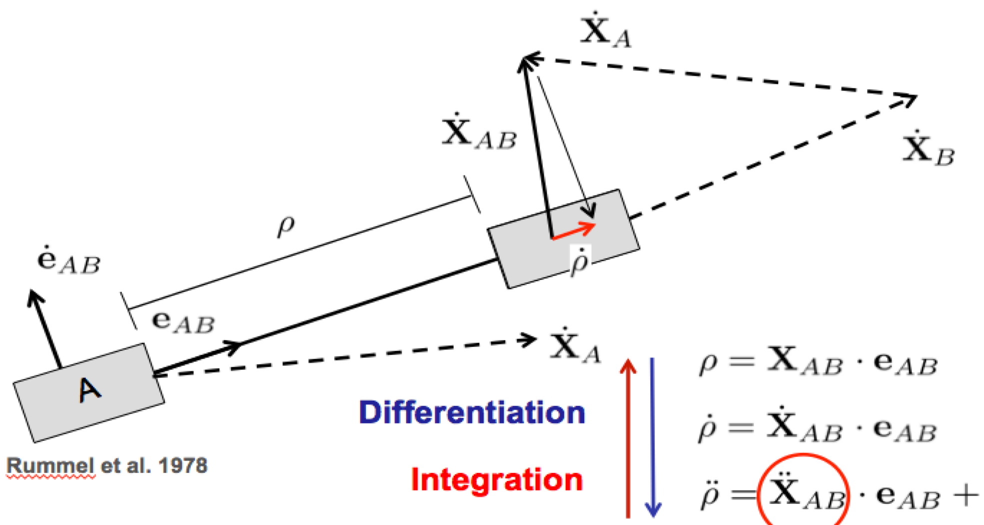
Universität Bern · Université du Luxembourg · GFZ Helmholtz Centre Potsdam · TU Graz · Leibniz Universität Hannover · cnes · DLR · géode&cie · HORIZON 2020

The acceleration approach-an alternative way of processing GRACE data

- Direct Link between kinematics (obs) and dynamics (force)
- Existing GRACE gravity field solutions based on acceleration approach:
  - Mean acceleration approach
    - DMT-1 (Liu et al., 2010)
    - Tongji_Acc RL01 (Chen et al., 2015)
  - Point-wise acceleration approach
    - WHIGG-GEGM01S (Cheng et al., 2012)
- An alternative way: connect the range accelerations with gradient of the gravitational potential

Recall the functional model of the classical acceleration approach

For the GRACE ll-SST case,



$$\rho = \mathbf{x}_{AB} \cdot \mathbf{e}_{AB}$$

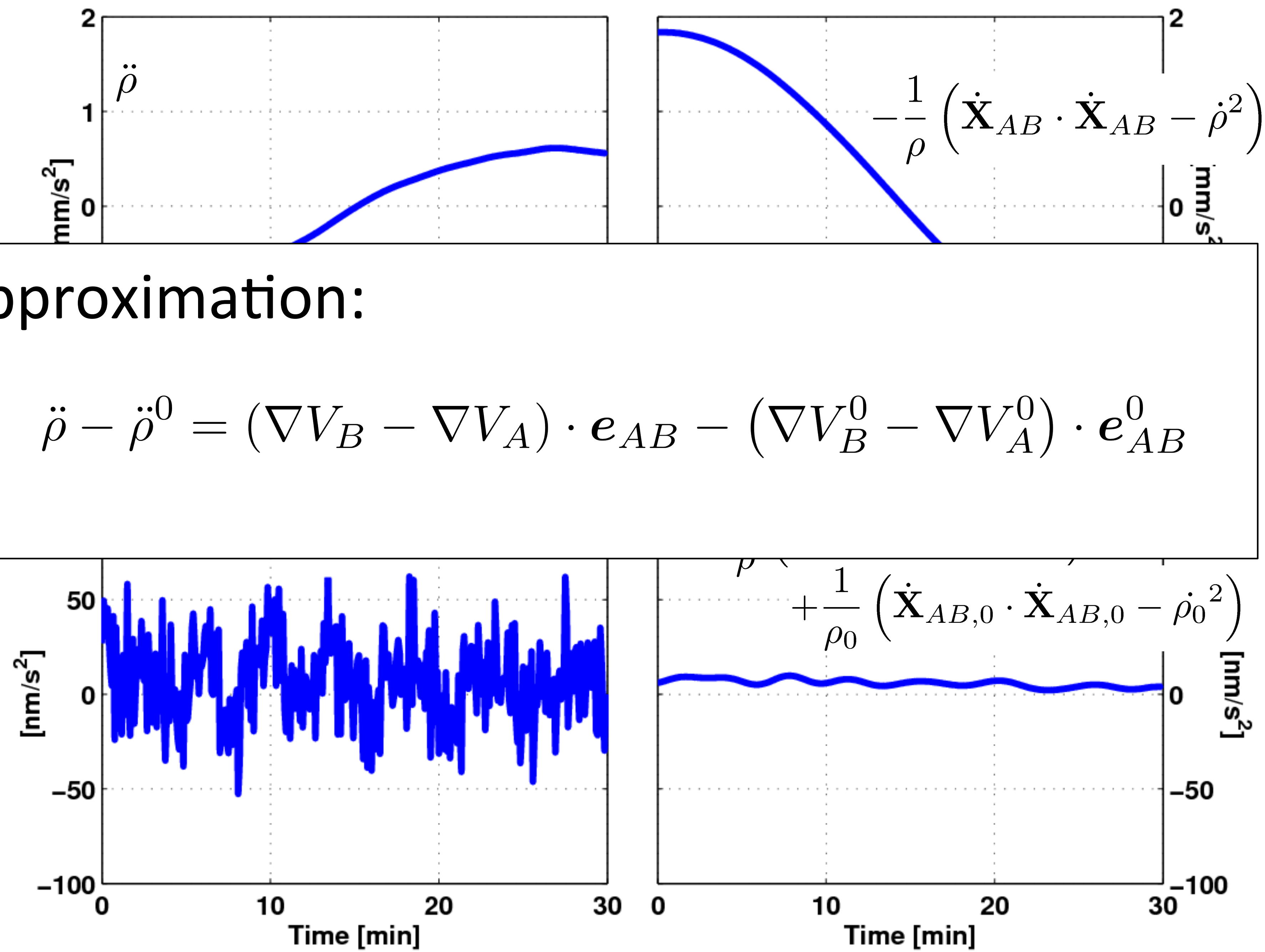
$$\dot{\rho} = \dot{\mathbf{x}}_{AB} \cdot \mathbf{e}_{AB}$$

$$\ddot{\rho} = \ddot{\mathbf{x}}_{AB} \cdot \mathbf{e}_{AB} + \dot{\mathbf{x}}_{AB} \cdot \dot{\mathbf{e}}_{AB}$$

$$= \nabla V_{AB}$$

$$\nabla V_{AB} \cdot \mathbf{e}^{\mathrm{a}}_{\mathrm{AB}} = \ddot{\rho} - \frac{1}{\rho}\left(\dot{\mathbf{x}}_{\mathrm{AB}} \cdot \dot{\mathbf{x}}_{\mathrm{AB}} - \dot{\rho}^2\right)$$

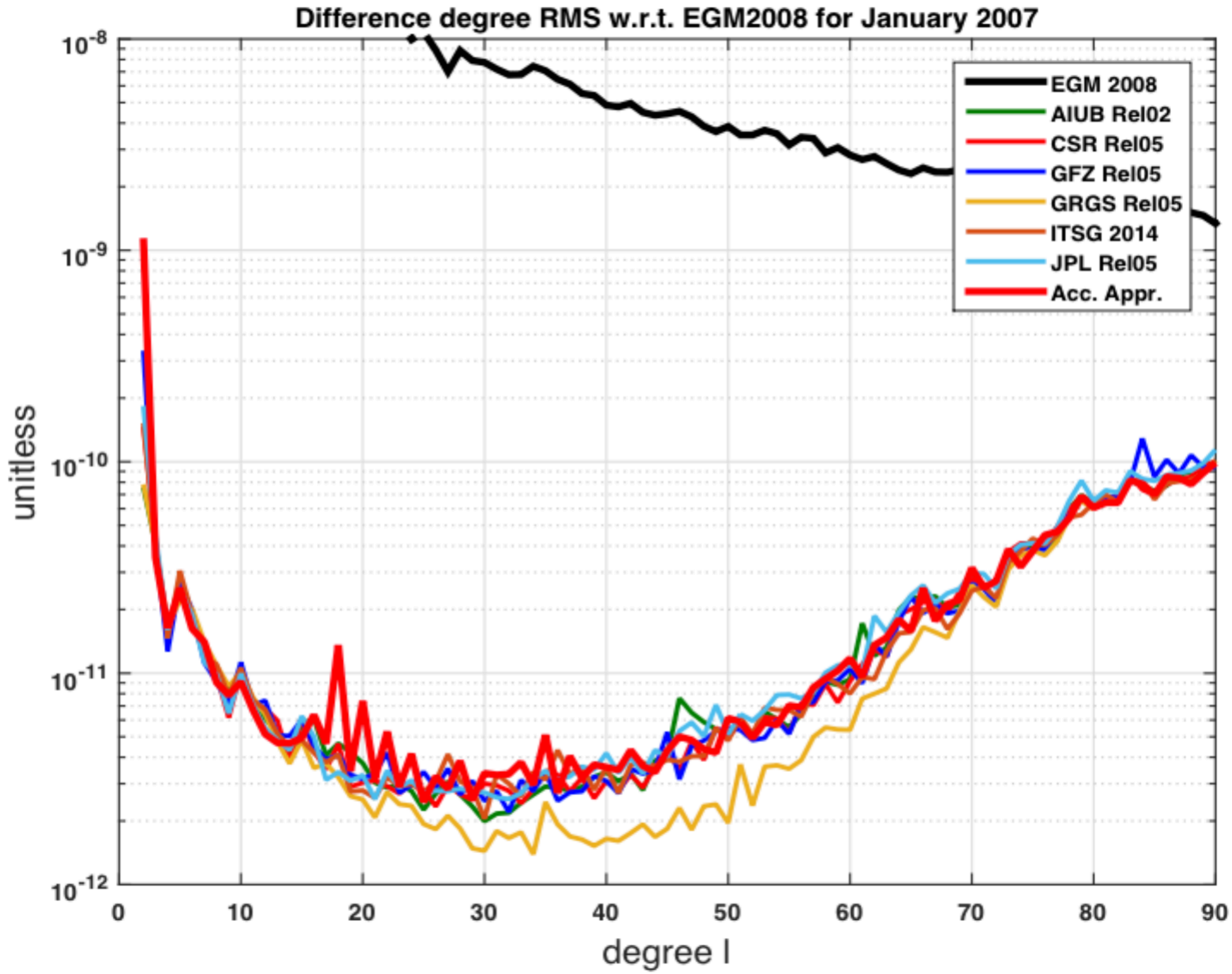
Approach: $\nabla V \cdot \boldsymbol{e}_{AB} = \ddot{\rho} - \frac{1}{\rho}\left(\|\dot{\mathbf{X}}_{AB}\|^2 - \dot{\rho}^2\right)$



Approximation:

$$\ddot{\rho} - \ddot{\rho}^0 = (\nabla V_B - \nabla V_A)\cdot \boldsymbol{e}_{AB} - \left(\nabla V_B^0 - \nabla V_A^0\right)\cdot \boldsymbol{e}_{AB}^0$$

Approximate solution



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Refinement (Rigorous solution)

- Full expression:

$$\ddot{\rho} - \ddot{\rho}^0 = (\nabla V_B - \nabla V_A) \cdot \boldsymbol{e}_{AB} - \left(\nabla V_B^0 - \nabla V_A^0\right) \cdot \boldsymbol{e}_{AB}^0 + \frac{1}{\rho}\left(\dot{\boldsymbol{X}}_{AB} \cdot \dot{\boldsymbol{X}}_{AB} - \dot{\rho}^2\right) - \frac{1}{\rho^0}\left(\dot{\boldsymbol{X}}^0_{AB} \cdot \dot{\boldsymbol{X}}^0_{AB} - \left(\dot{\rho}^0\right)^2\right)$$

- Applying linearization yields:

$$\nabla V_{AB} \cdot \boldsymbol{e}_{AB} - \nabla V_{AB}^0 \cdot \boldsymbol{e}_{AB}^0 = \sum_i \frac{\partial f_1}{\partial s_i}\Delta s_i + \sum_i \frac{\partial f_2}{\partial s_i}\Delta s_i + \hbar^2$$

$$\frac{1}{\rho}\|\dot{\boldsymbol{X}}_{AB}\|^2 - \frac{1}{\rho^0}\|\dot{\boldsymbol{X}}^0_{AB}\|^2 = \sum_i \frac{\partial g_1}{\partial s_i}\Delta s_i + \hbar^2$$

$$-\frac{\dot{\rho}^2}{\rho} + \frac{\left(\dot{\rho}^0\right)^2}{\rho^0} = \sum_i \frac{\partial g_2}{\partial s_i}\Delta s_i + \hbar^2$$

- f1 is the relative gravity vector and f2 is the line of sight unit vector

Refinement (Rigorous solution)

- Partial derivatives for f1 toward Clm and Slm can be derived analytically:

$$\frac{\partial f_1}{\partial \overline{C}_{lm}} = \frac{\partial(\partial V_B/\partial x_E)}{\partial \overline{C}_{lm}} \cdot e^0_{AB,x_E} + \frac{\partial(\partial V_B/\partial y_E)}{\partial \overline{C}_{lm}} \cdot e^0_{AB,y_E} + \frac{\partial(\partial V_B/\partial z_E)}{\partial \overline{C}_{lm}} \cdot e^0_{AB,z_E} - \frac{\partial(\partial V_A/\partial x_E)}{\partial \overline{C}_{lm}} \cdot e^0_{AB,x_E} - \frac{\partial(\partial V_A/\partial y_E)}{\partial \overline{C}_{lm}} \cdot e^0_{AB,y_E} - \frac{\partial(\partial V/\partial z_E)}{\partial \overline{C}_{lm}} \cdot e^0_{AB,z_E}$$

$$\frac{\partial f_1}{\partial \overline{S}_{lm}} = \frac{\partial(\partial V_B/\partial x_E)}{\partial \overline{S}_{lm}} \cdot e^0_{AB,x_E} + \frac{\partial(\partial V_B/\partial y_E)}{\partial \overline{S}_{lm}} \cdot e^0_{AB,y_E} + \frac{\partial(\partial V_B/\partial z_E)}{\partial \overline{S}_{lm}} \cdot e^0_{AB,z_E} - \frac{\partial(\partial V_A/\partial x_E)}{\partial \overline{S}_{lm}} \cdot e^0_{AB,x_E} - \frac{\partial(\partial V_A/\partial y_E)}{\partial \overline{S}_{lm}} \cdot e^0_{AB,y_E} - \frac{\partial(\partial V/\partial z_E)}{\partial \overline{S}_{lm}} \cdot e^0_{AB,z_E}$$

- All the other partials are formed by chain rule and linked to the partial derivatives of position and velocity of each satellite toward the unknowns

- Variational equations need to be solved! More work than considering range/range rate observations because of g1 and g2

Variation of constant approach (Jaeggi, 2007)

- Variational equations for the initial conditions (homogeneous solution):

$$\frac{d}{dt}\begin{bmatrix} \frac{\partial x}{\partial x_0} & \frac{\partial x}{\partial y_0} & \cdots & \frac{\partial x}{\partial \dot{y}_0} & \frac{\partial x}{\partial \dot{z}_0} \\ \frac{\partial y}{\partial x_0} & \frac{\partial y}{\partial y_0} & \cdots & \frac{\partial y}{\partial \dot{y}_0} & \frac{\partial y}{\partial \dot{z}_0} \\ \frac{\partial z}{\partial x_0} & \frac{\partial z}{\partial y_0} & \cdots & \frac{\partial z}{\partial \dot{y}_0} & \frac{\partial z}{\partial \dot{z}_0} \\ \frac{\partial \dot{x}}{\partial x_0} & \frac{\partial \dot{x}}{\partial y_0} & \cdots & \frac{\partial \dot{x}}{\partial \dot{y}_0} & \frac{\partial \dot{x}}{\partial \dot{z}_0} \\ \frac{\partial \dot{y}}{\partial x_0} & \frac{\partial \dot{y}}{\partial y_0} & \cdots & \frac{\partial \dot{y}}{\partial \dot{y}_0} & \frac{\partial \dot{y}}{\partial \dot{z}_0} \\ \frac{\partial \dot{z}}{\partial x_0} & \frac{\partial \dot{z}}{\partial y_0} & \cdots & \frac{\partial \dot{z}}{\partial \dot{y}_0} & \frac{\partial \dot{z}}{\partial \dot{z}_0} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{\partial^2 V}{\partial x^2} & \frac{\partial^2 V}{\partial x \partial y} & \frac{\partial^2 V}{\partial x \partial z} & 0 & 0 & 0 \\ \frac{\partial^2 V}{\partial y \partial x} & \frac{\partial^2 V}{\partial y^2} & \frac{\partial^2 V}{\partial y \partial z} & 0 & 0 & 0 \\ \frac{\partial^2 V}{\partial z \partial x} & \frac{\partial^2 V}{\partial z \partial y} & \frac{\partial^2 V}{\partial z^2} & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial x}{\partial x_0} & \frac{\partial x}{\partial y_0} & \cdots & \frac{\partial x}{\partial \dot{y}_0} & \frac{\partial x}{\partial \dot{z}_0} \\ \frac{\partial y}{\partial x_0} & \frac{\partial y}{\partial y_0} & \cdots & \frac{\partial y}{\partial \dot{y}_0} & \frac{\partial y}{\partial \dot{z}_0} \\ \frac{\partial z}{\partial x_0} & \frac{\partial z}{\partial y_0} & \cdots & \frac{\partial z}{\partial \dot{y}_0} & \frac{\partial z}{\partial \dot{z}_0} \\ \frac{\partial \dot{x}}{\partial x_0} & \frac{\partial \dot{x}}{\partial y_0} & \cdots & \frac{\partial \dot{x}}{\partial \dot{y}_0} & \frac{\partial \dot{x}}{\partial \dot{z}_0} \\ \frac{\partial \dot{y}}{\partial x_0} & \frac{\partial \dot{y}}{\partial y_0} & \cdots & \frac{\partial \dot{y}}{\partial \dot{y}_0} & \frac{\partial \dot{y}}{\partial \dot{z}_0} \\ \frac{\partial \dot{z}}{\partial x_0} & \frac{\partial \dot{z}}{\partial y_0} & \cdots & \frac{\partial \dot{z}}{\partial \dot{y}_0} & \frac{\partial \dot{z}}{\partial \dot{z}_0} \end{bmatrix}$$

- Variation of constant (inhomogeneous solution):

$$\boldsymbol{\alpha}_{p_i}(t) = \int_{t_0}^{t} \Phi^{-1}(\tau) \cdot \frac{\partial \mathbf{h}(\tau)}{\partial p_i} d\tau$$

$$\boldsymbol{\phi}_{p_i}(t) = \Phi(t) \cdot \boldsymbol{\alpha}_{p_i}(t) \, .$$

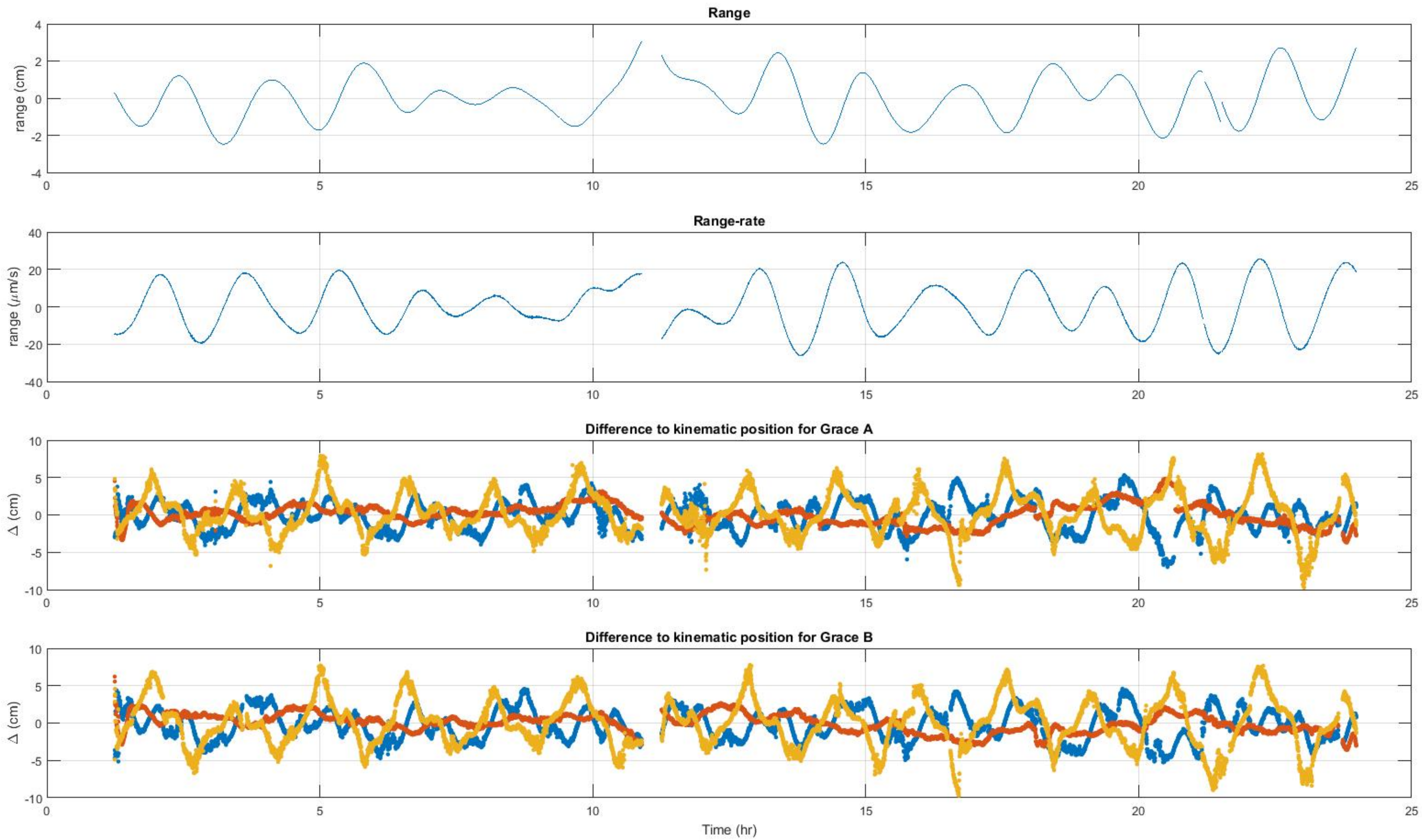
- Partial derivatives toward the unknowns could then be established via chain rule using the above solutions

Current implementation status at UL

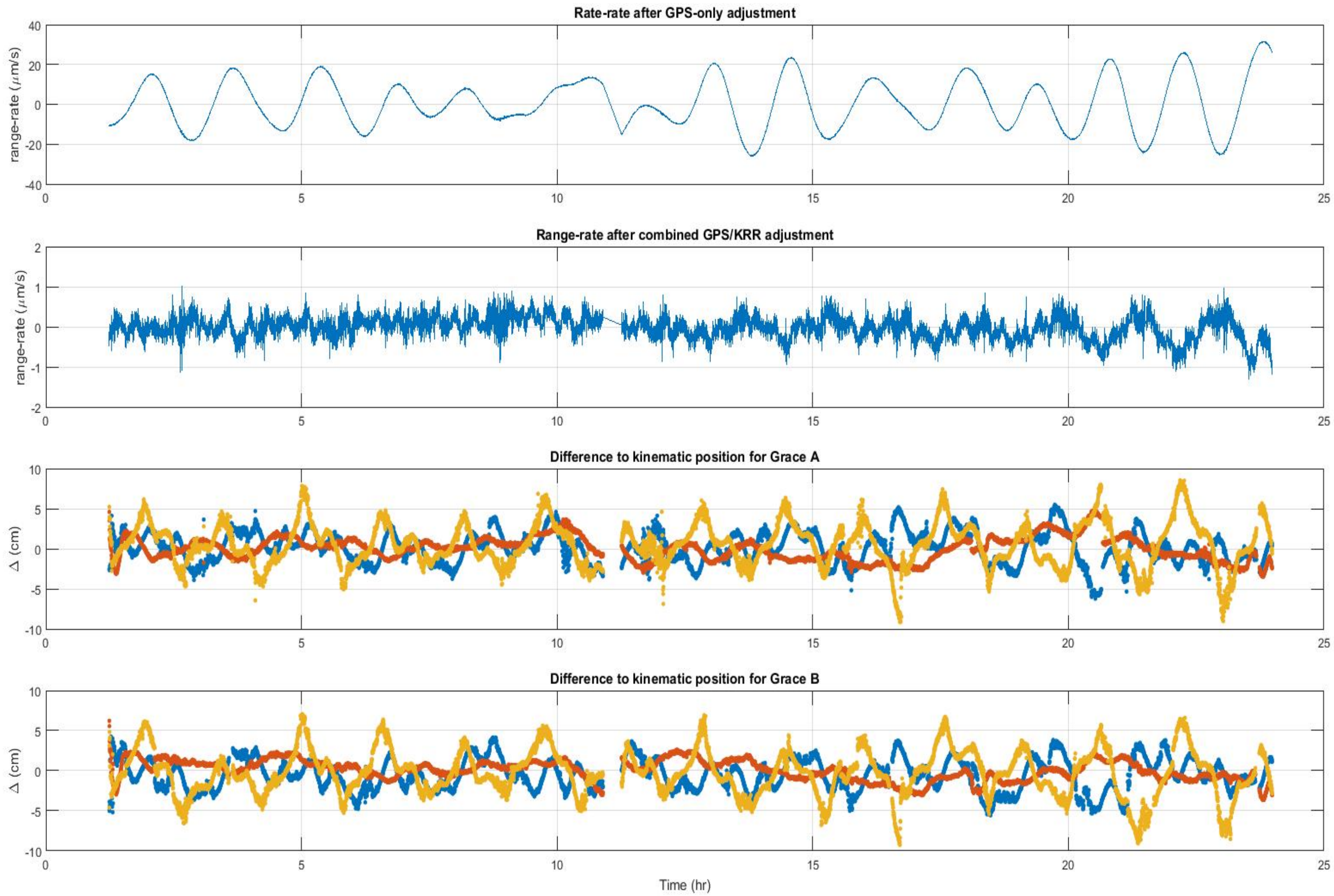
- **Codes for the rigorous approach have almost been finished. They are now under the test stage**

- **A priori orbit of Grace A and B for later gravity field determination (iteration)**
  - **Method: CMA, totally follow AIUB (Beutler et al., 2010)**
  - **Input data: LIB GRAZ Kin, Nav, KRR, accelerometer and star camera data**
  - **Arc-length: 24 hours**
  - **Parameterization: 6 initial state vectors per arc, empirical piece-wise constant acceleration every 15min per axis per arc, accelerometer scale and bias per axis per day**
  - **Constraints: pos 0.3, vel 0.03, emp acc 3e-9, acc scale 1e-4, acc bias 1e-8, during combined adjustment, p1 and p2 applied with different constraints: 3e-9 and 3e-11**
  - **Scaling ratio of krr/gps is calculated ∂krr2/ ∂gps2**
  - **Background model:**
    - Tide-system: Tide-free
    - Goco05s is used as the a priori gravity model
    - EGM2008 is used for correcting the impact of coefficients from maximum degree (60 or 90) to 250
    - Solid earth and pole tide: IERS 2003
    - Ocean pole tide: Desai model complete to degree and order 120
    - Ocean tide: EOT11a, degree and order 120
    - Atmospheric tides: only S2, degree and order 8 (Biancale & Bode 2006, GFZ from ECMWF model)
    - High frequency atmosphere and ocean mass redistribution: AOD1B RL05, complete to degree and order 100 (Flechtner & Dobslaw 2013), remove the S2 part
    - Body tides: moon, sun, mercury, venus, mars, Jupiter, Saturn, Uranus, Neptune, DE421 used (Folkner et al 2008)
    - Relativistic effect: IERS 2010
    - Earth rotation: IERS 2010

Orbit adjustment result for Grace A and B on 2006-01-01



KRR contribution for Grace A and B on 2006-01-01



Current implementation status at UL

- **Solve for orbit and the gravity field parameters using our rigorous acceleration approach(no iteration)**

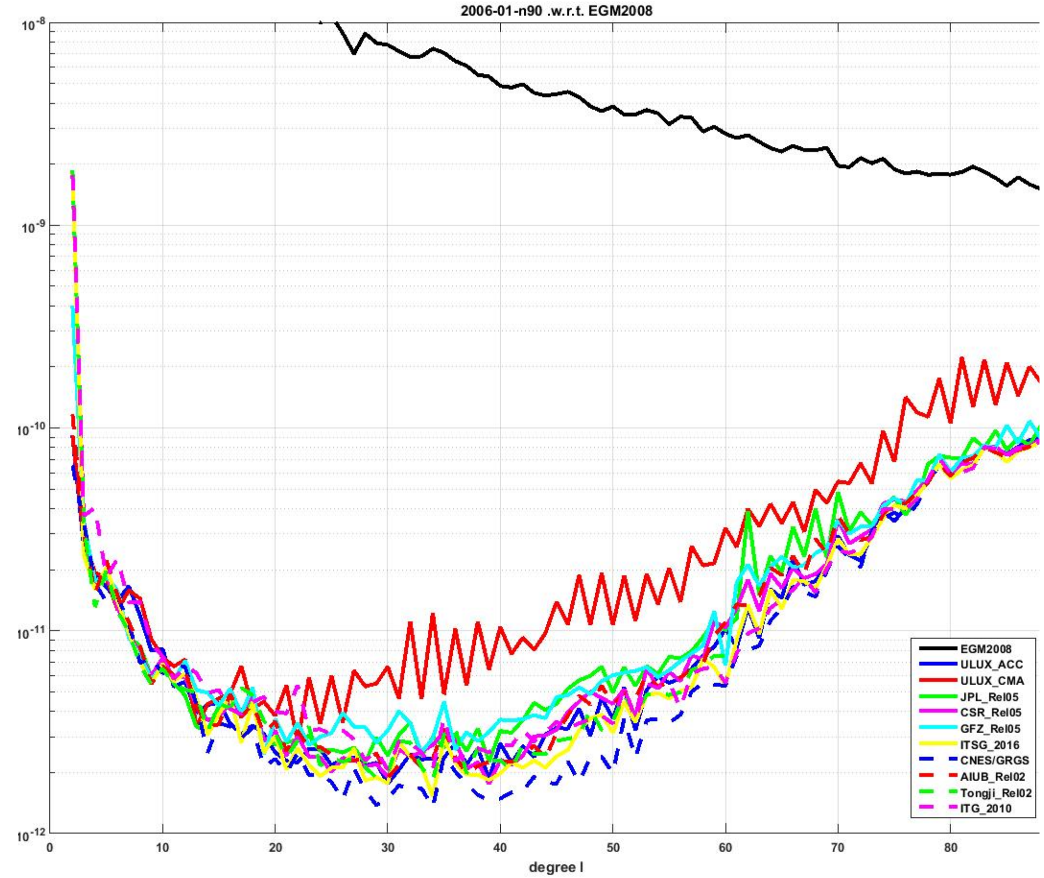
- **Only ll-SST considered so far, the observation equation is:**

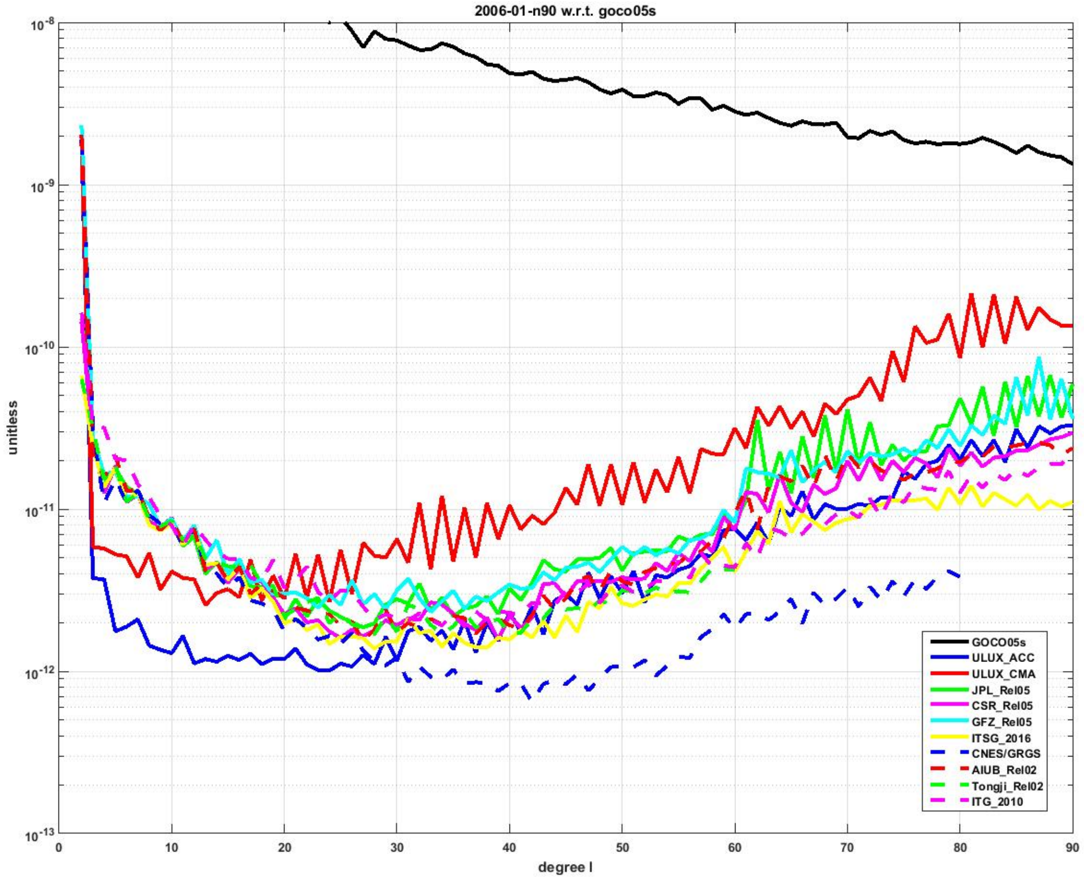
$$\ddot{\rho} - \ddot{\rho}^0 = (\nabla V_B - \nabla V_A) \cdot \boldsymbol{e}_{AB} - \left(\nabla V_B^0 - \nabla V_A^0\right) \cdot \boldsymbol{e}_{AB}^0 + \frac{1}{\rho}\left(\dot{\boldsymbol{X}}_{AB} \cdot \dot{\boldsymbol{X}}_{AB} - \dot{\rho}^2\right) - \frac{1}{\rho^0}\left(\dot{\boldsymbol{X}}_{AB}^0 \cdot \dot{\boldsymbol{X}}_{AB}^0 - \left(\dot{\rho}^0\right)^2\right)$$

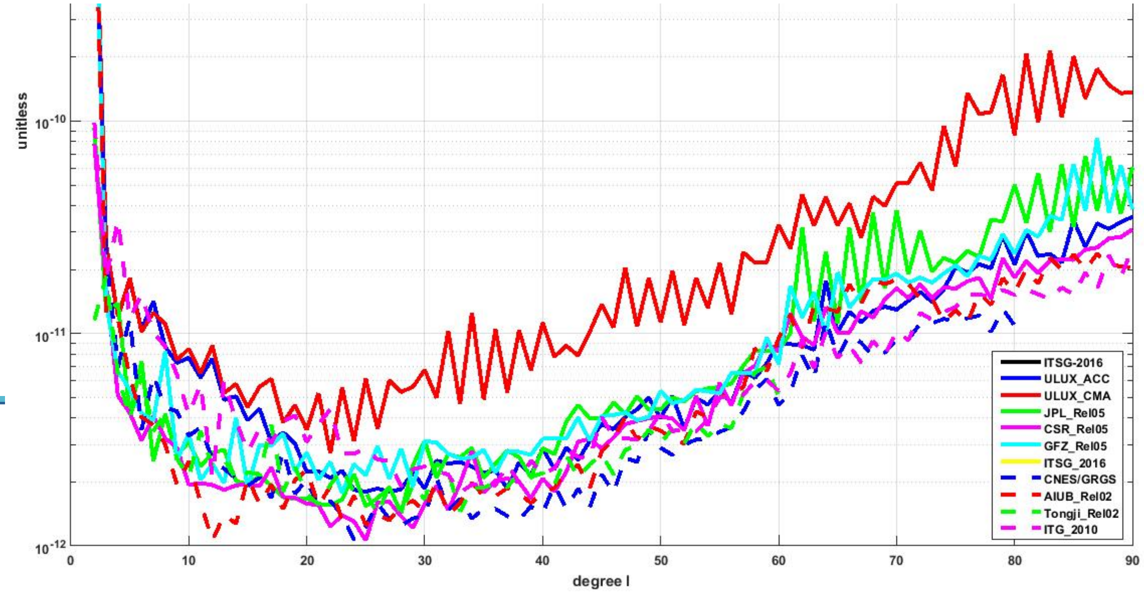
- **Position and range-rate also needed to form normal equations**

- **The rigorous acceleration approach is just another implementation of the variational equations**

Test gravity coefficient results for 01/2006 until degree 90







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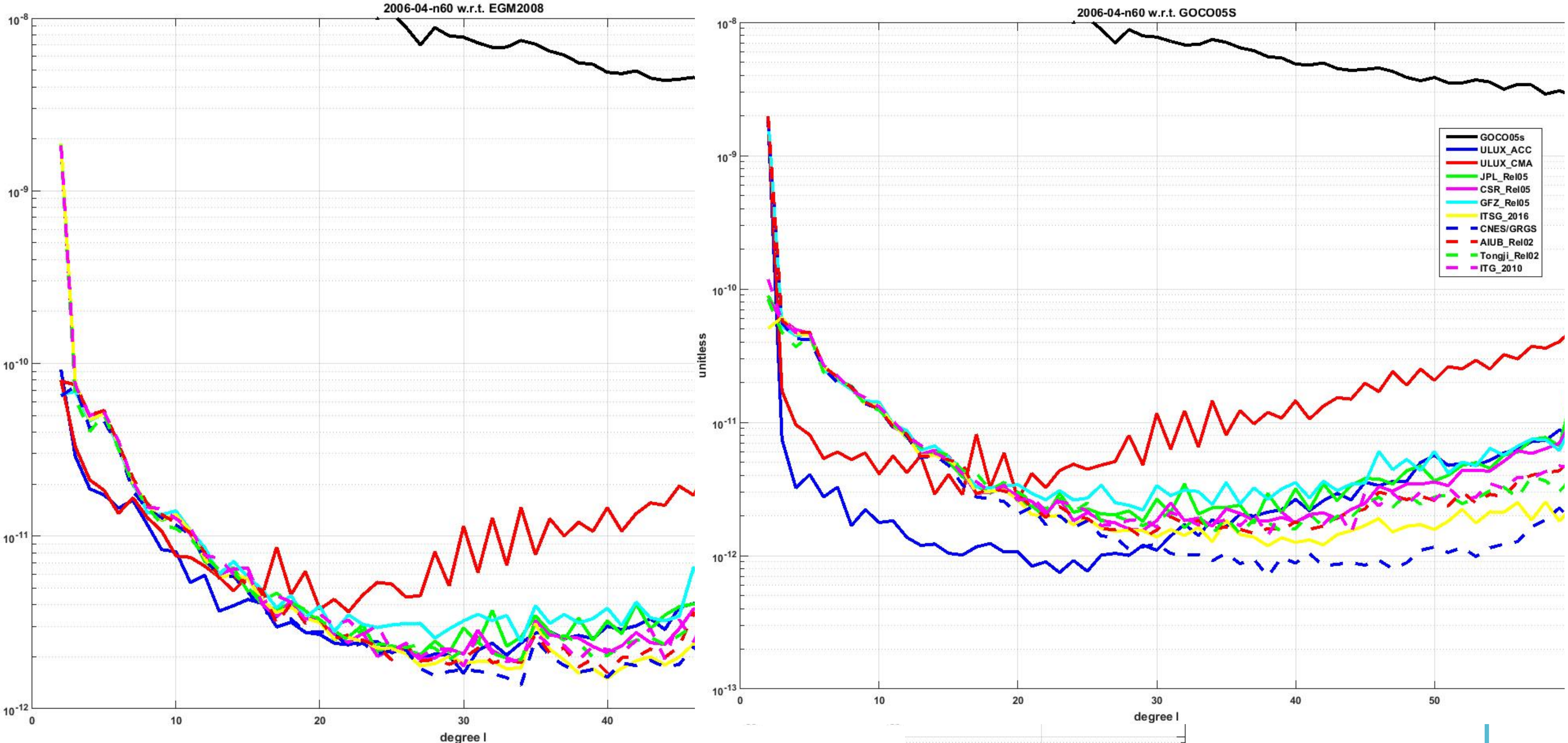
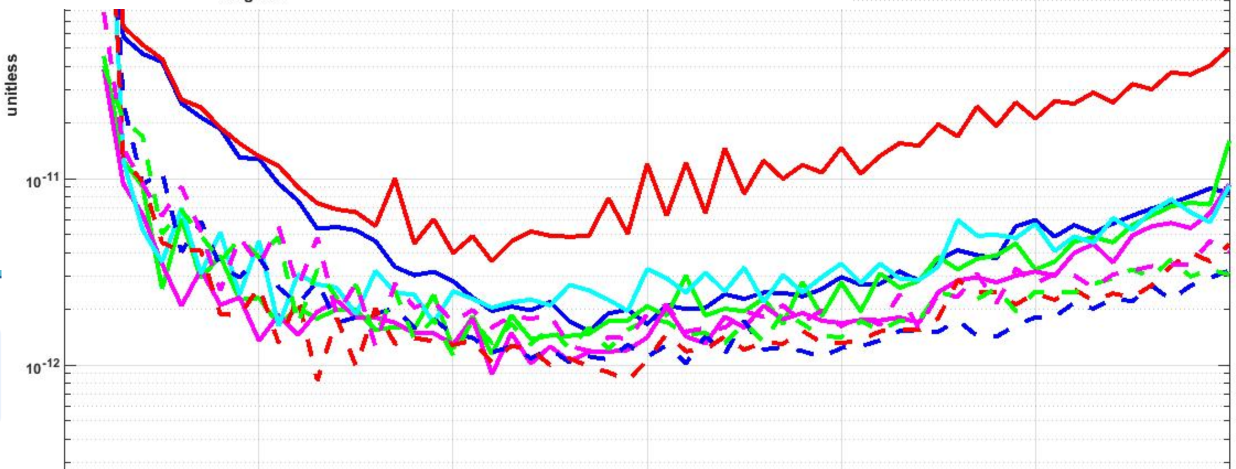
Test gravity coefficient results for 04/2006 until degree 60





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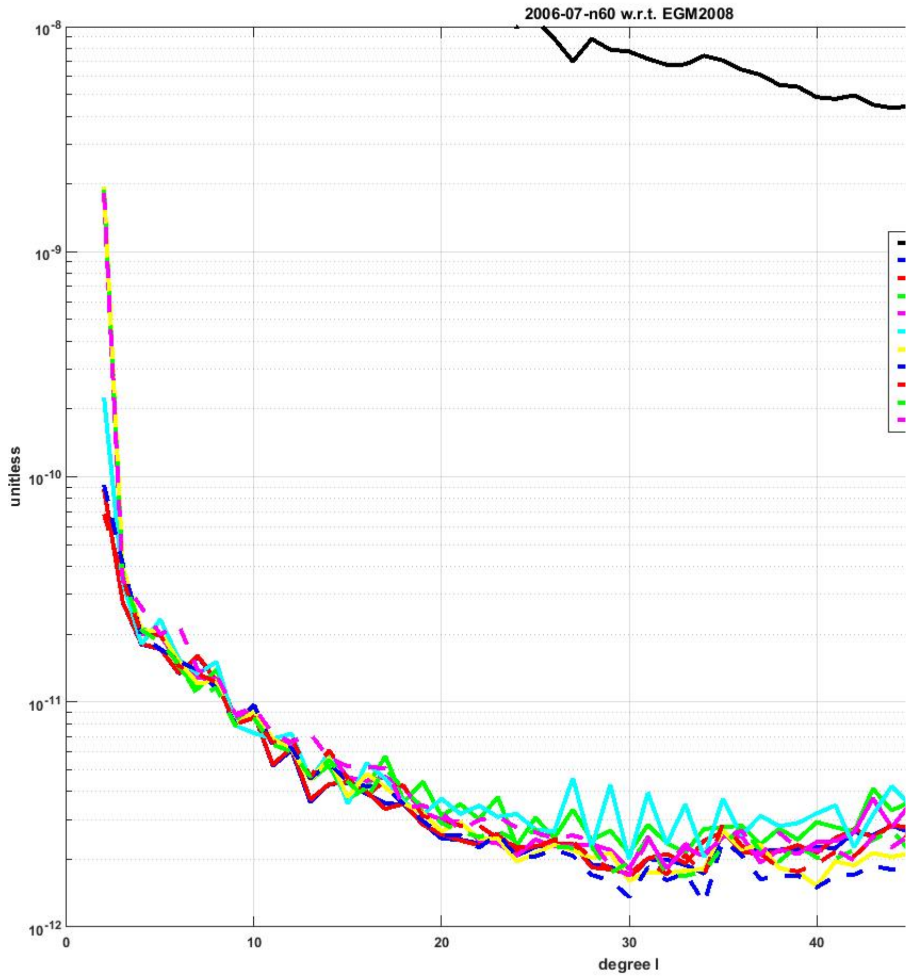
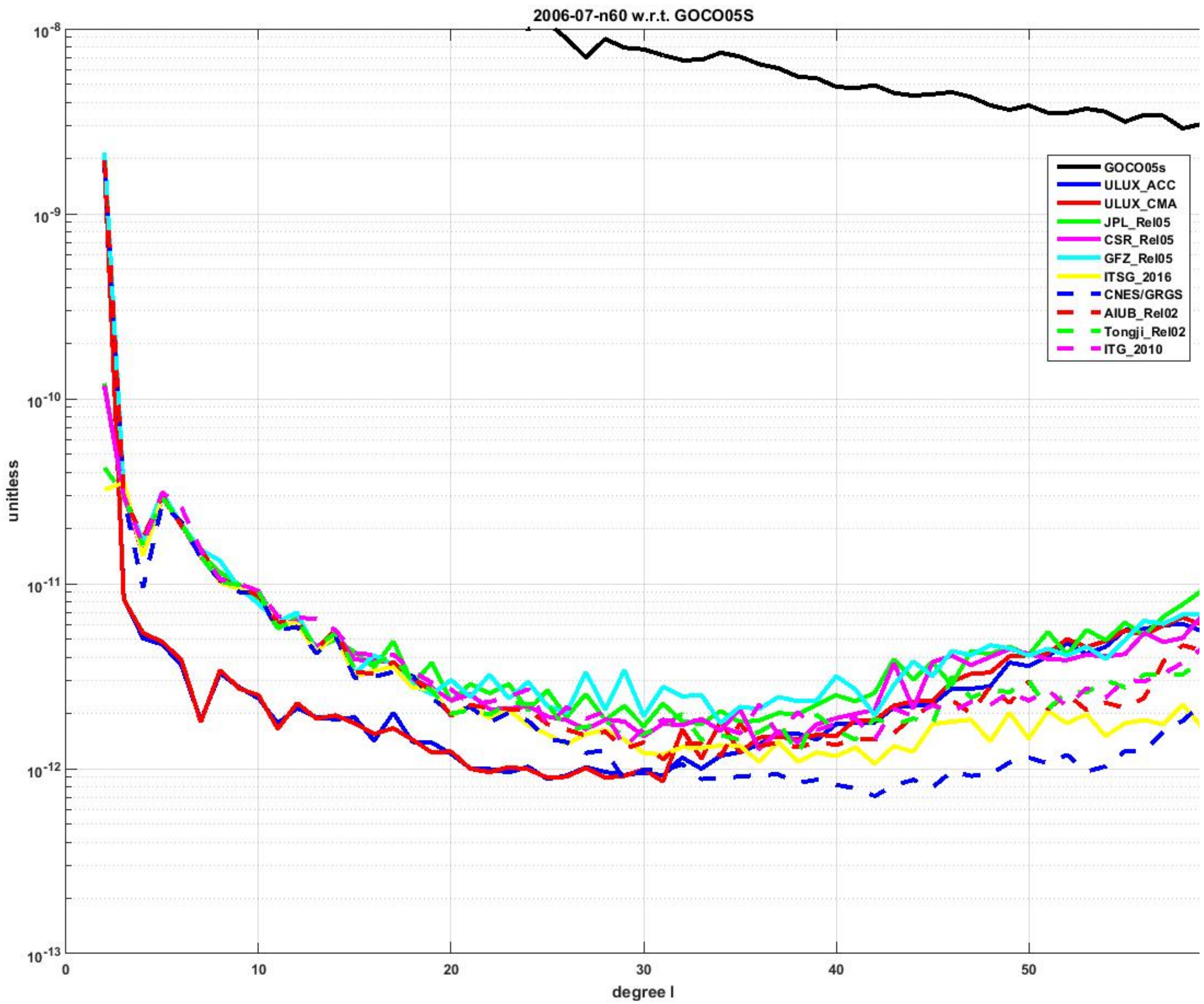
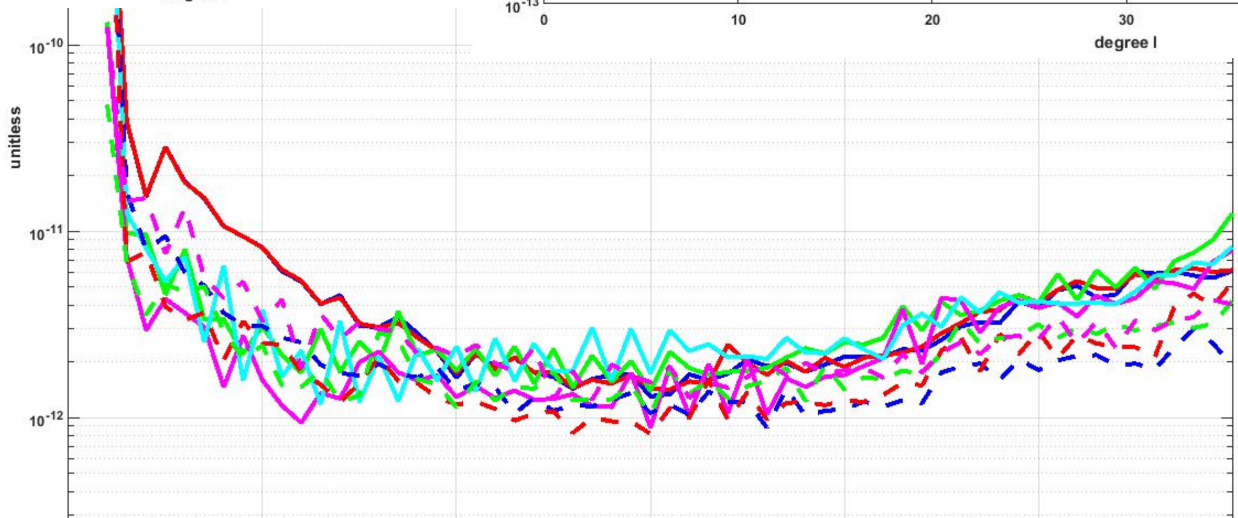
Test gravity coefficient results for 07/2006 until degree 60







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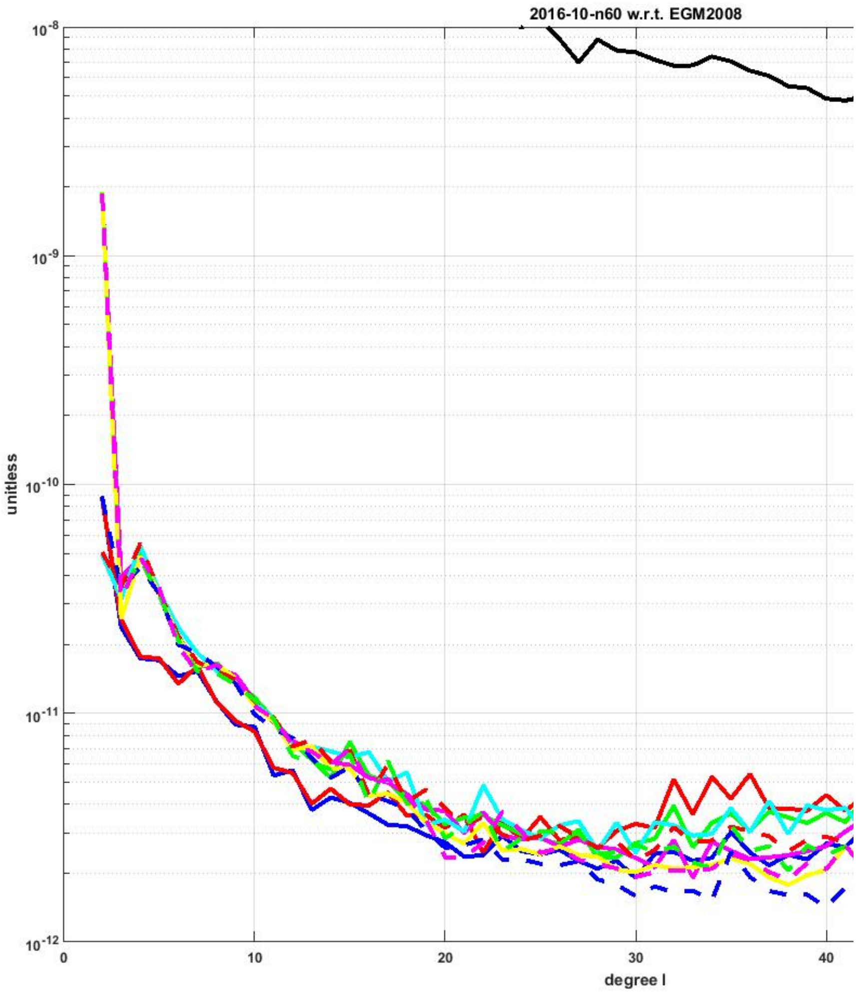
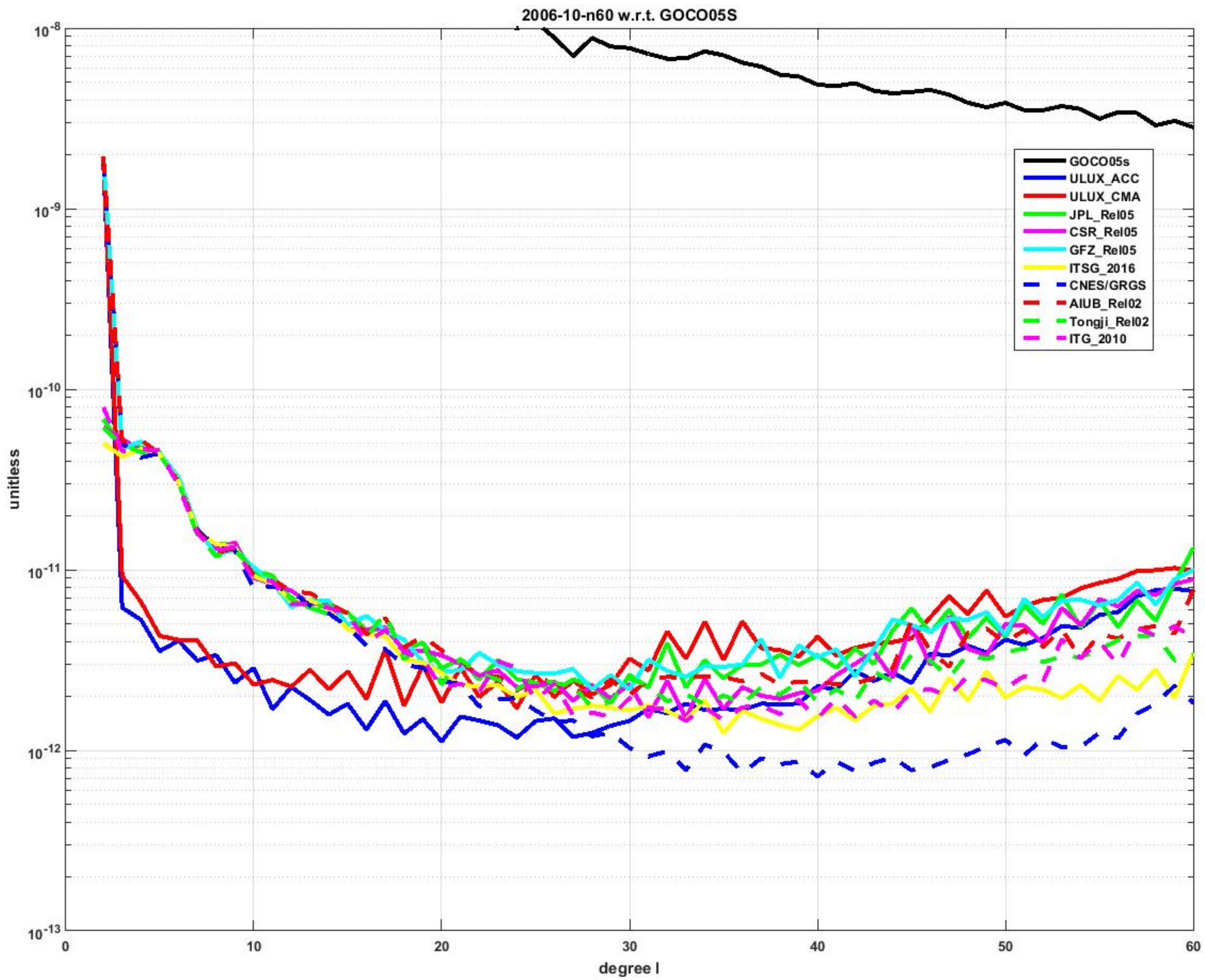
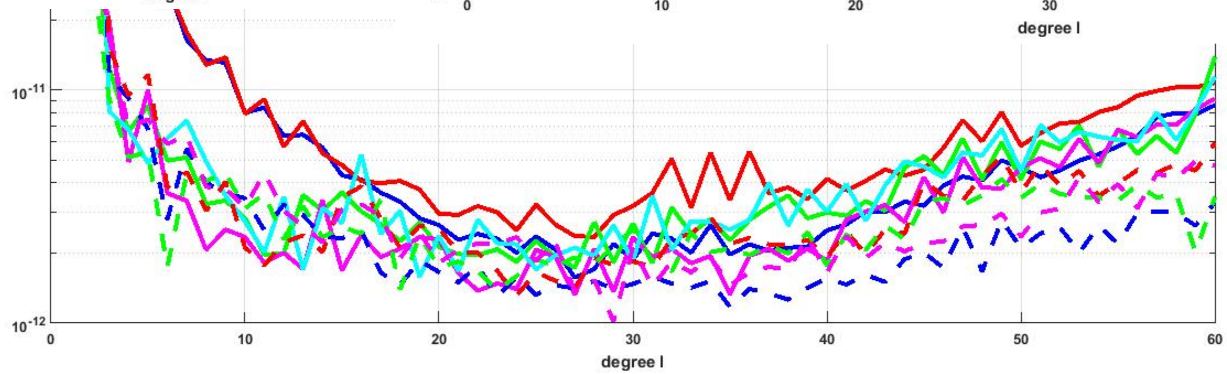
Test gravity coefficient results for 10/2006 until degree 60







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Work plan in 2007

- The implementation has been improved and we are getting more closer.
- Still issues to be solved
  - Some of the orbit are not good enough
    - Shorter arc length, piece wise linear acc,…
    - Better integrator
    - Parameterization
    - …
  - Solution biased toward GOCO05S
- More detailed analysis and comparison with the other analysis centers
  - Validation with hydrology
  - Validation with GPS leveling and displacement
- Good news is that ULUX is progressing, and we are confident to deliver full solution for 2006-2007 at the end of June

Thank you very much!