EGSIEM

European Gravity Service for Improved Emergency Management

Combination on Normal Equation Level

Ulrich Meyer (AIUB)

EGSIEM General Assembly

AIUB Bern

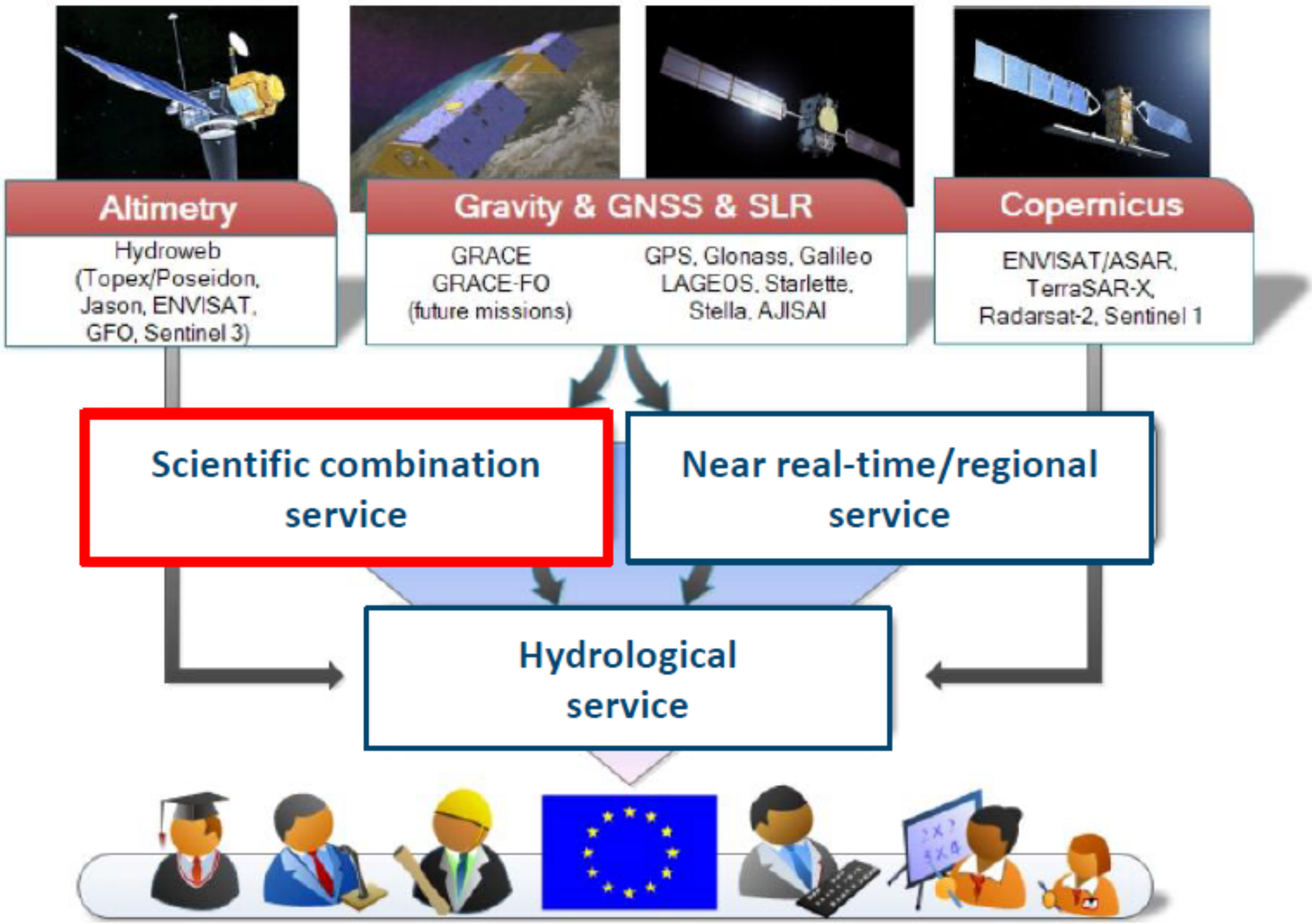
January 19 - 20, 2017

Universität Bern · Université du Luxembourg · GFZ Helmholtz Centre Potsdam · TU Graz Graz University of Technology · Leibniz Universität Hannover · cnes · DLR · géode & cie · Horizon2020

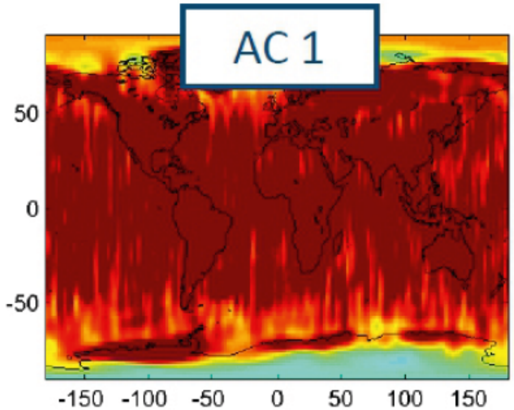
Contents

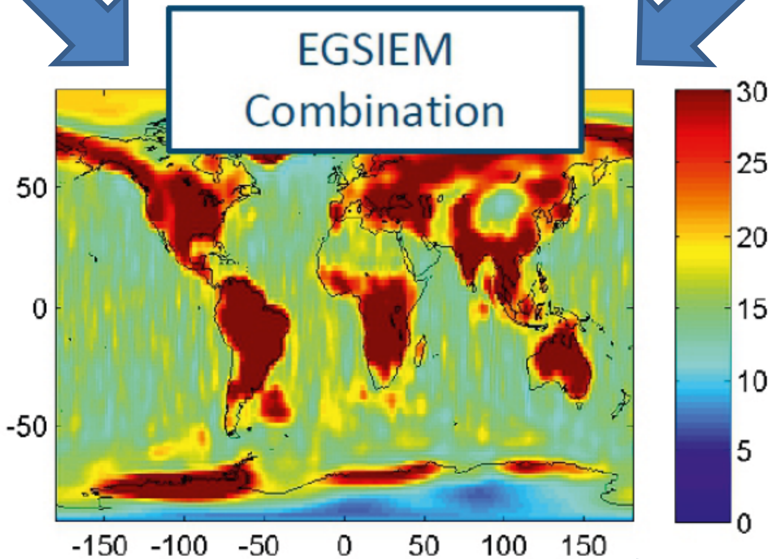
- Motivation for NEQ-combination
- Weighting schemes
- Combination results

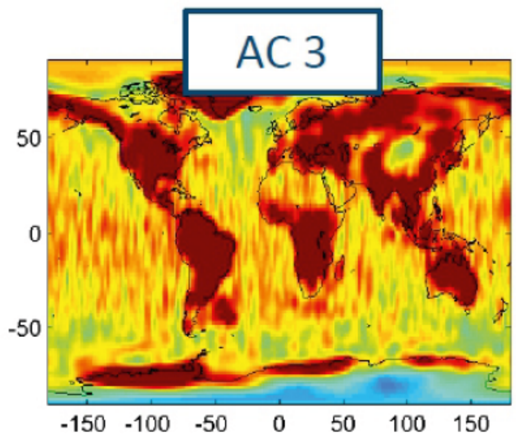
EGSIEM Project – Three services are beeing established

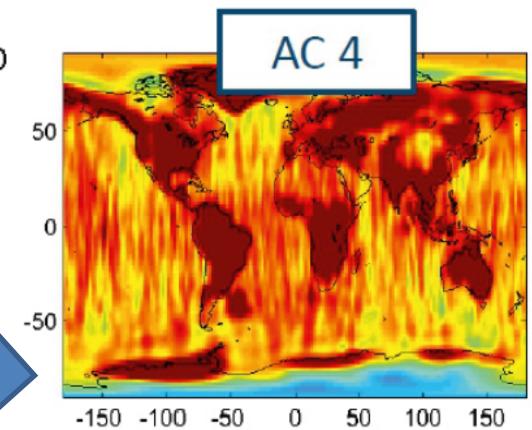


Scientific Combination Service









- **Only one product for the user**
- **Reduced noise**

Scientific Combination Service

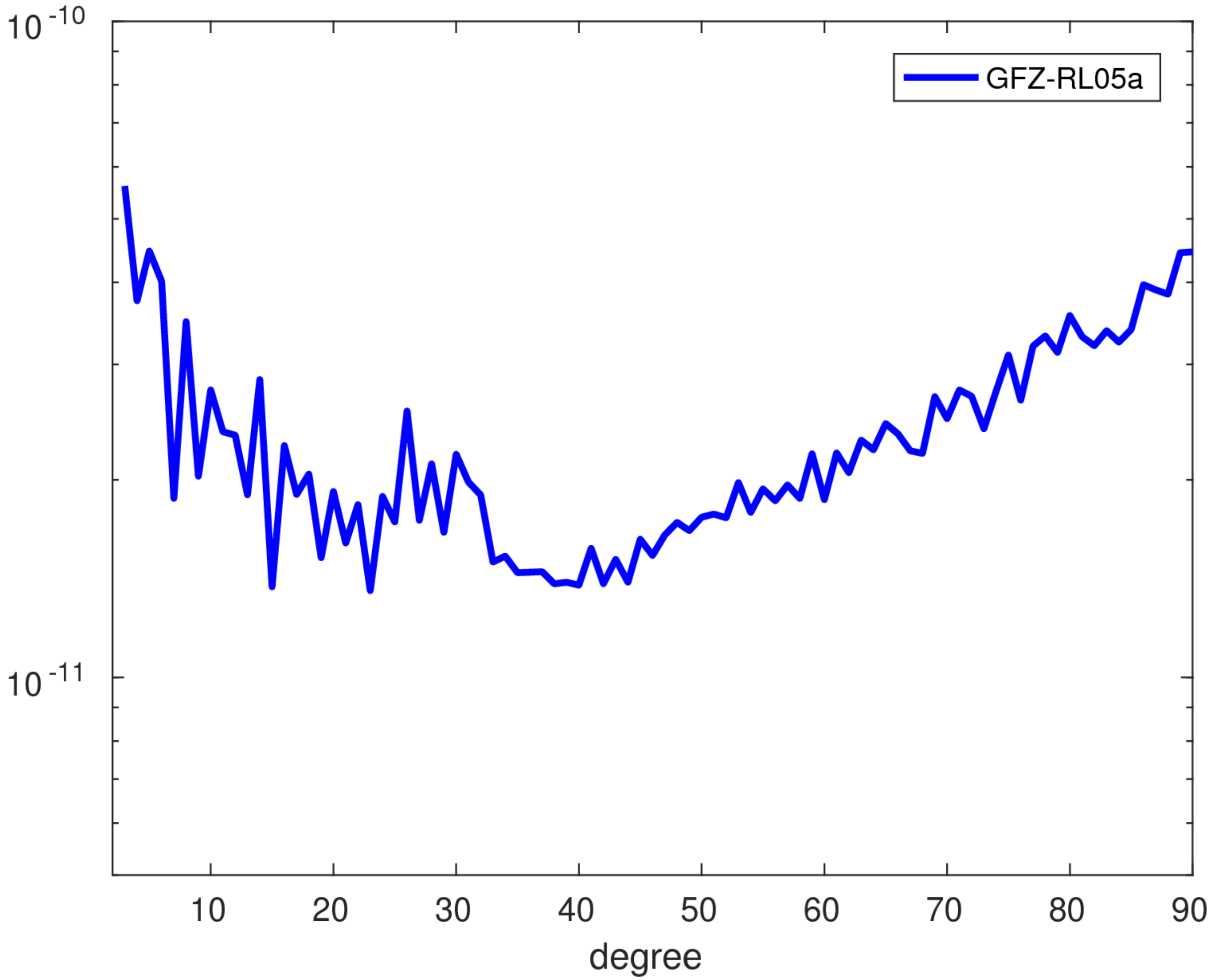
- The EGSIEM combination service provides monthly GRACE K-band gravity fields combined on solution / normal equation (NEQ) Level.
- To ensure consistency, a set of common standards for reference frame, Earth rotation, force model and satellite geometry were defined.
- EGSIEM lately was extended to also include SLR and GPS-only NEQs.

Why combine results based on the same observations?
Errors in GRACE monthly gravity fields are still dominated by analysis and background model noise, not observation noise!

Motivation for NEQ-Combination

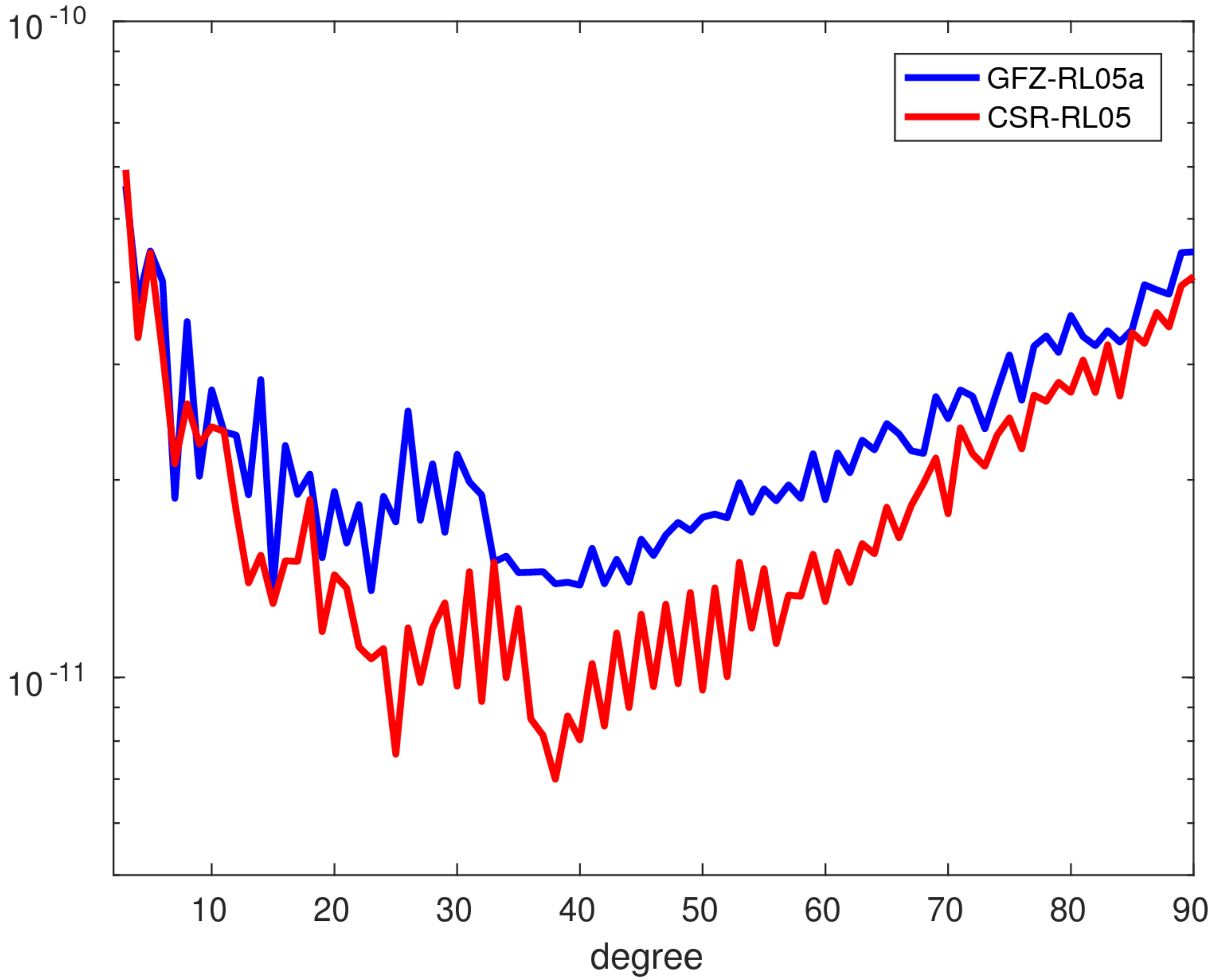
- Correlations are correctly taken into account, even with pre-eliminated parameters.
- In principle corrections are estimated for the original observations, not the intermediate individual model parameters.

Comparison to official solutions 2006/01



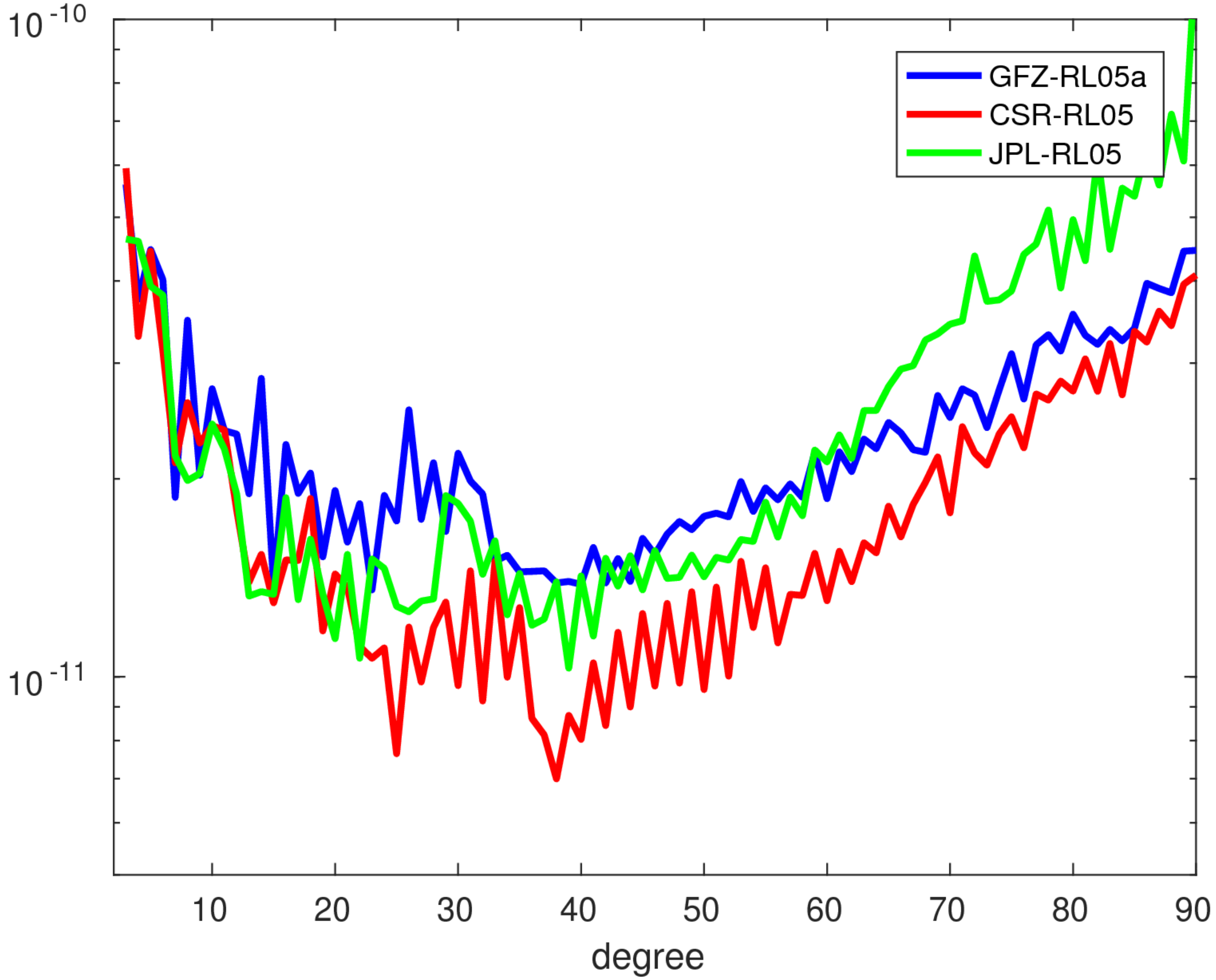
- Degree amplitudes of anomalies with respect to modeled secular and seasonal variations (based on ICGEM dataset).
- Only orders 0..29 are considered: evaluation of part of the spectrum that is determined meaningful.

Comparison to official solutions 2006/01



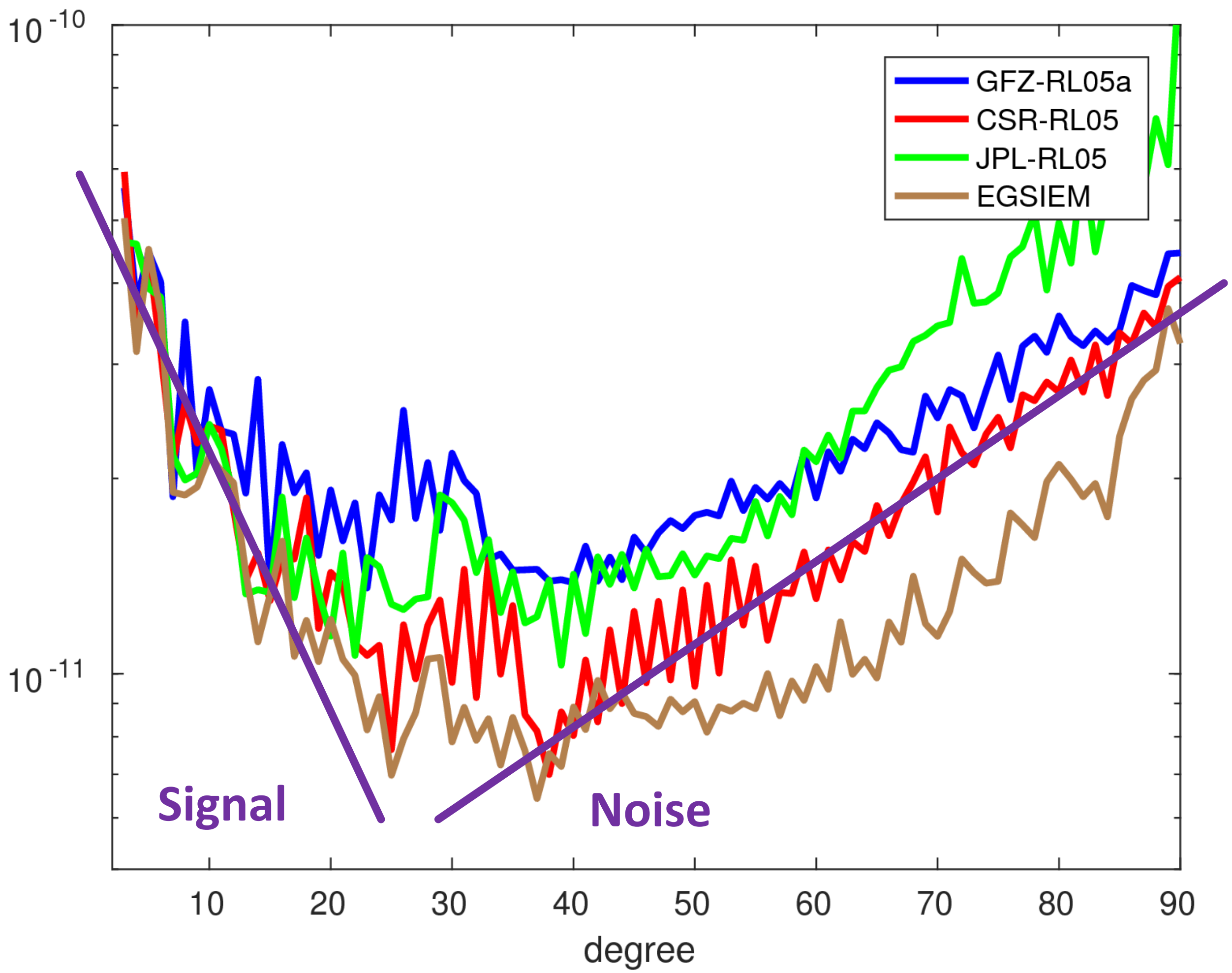
- Degree amplitudes of anomalies with respect to modeled secular and seasonal variations (based on ICGEM dataset).
- Only orders 0..29 are considered: evaluation of part of the spectrum that is determined meaningful.

Comparison to official solutions 2006/01



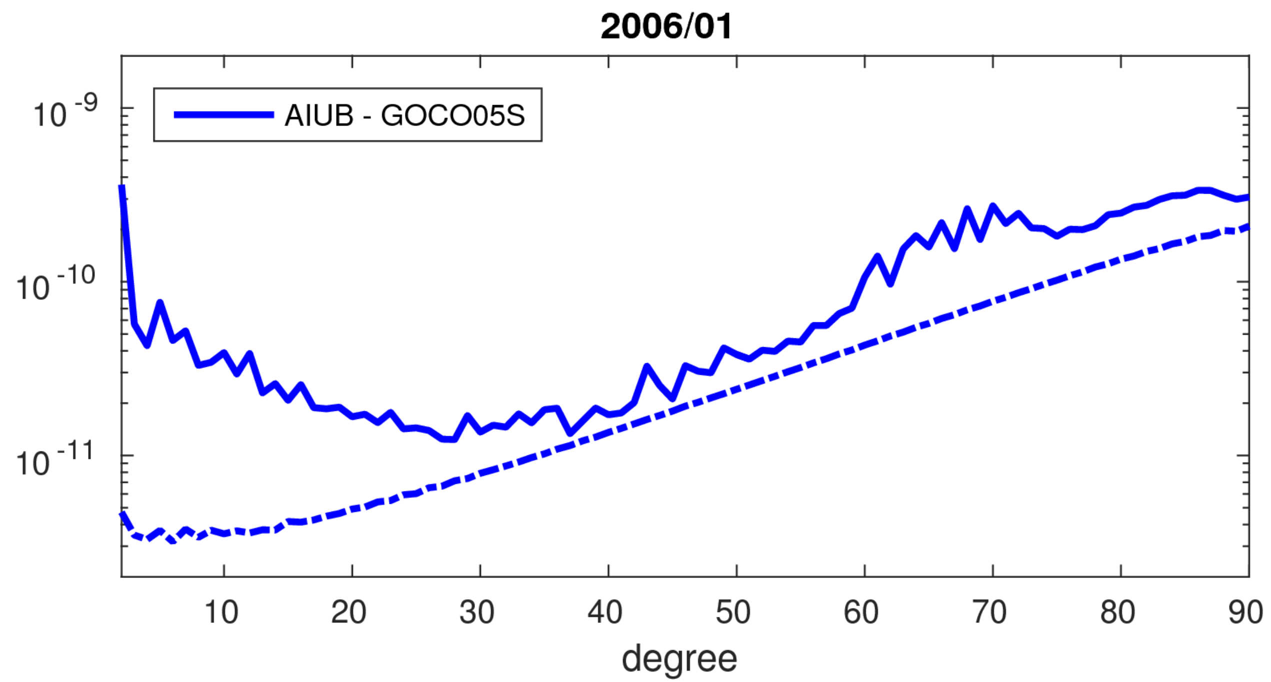
- Degree amplitudes of anomalies with respect to modeled secular and seasonal variations (based on ICGEM dataset).
- Only orders 0..29 are considered: evaluation of part of the spectrum that is determined meaningful.

Comparison to official solutions 2006/01



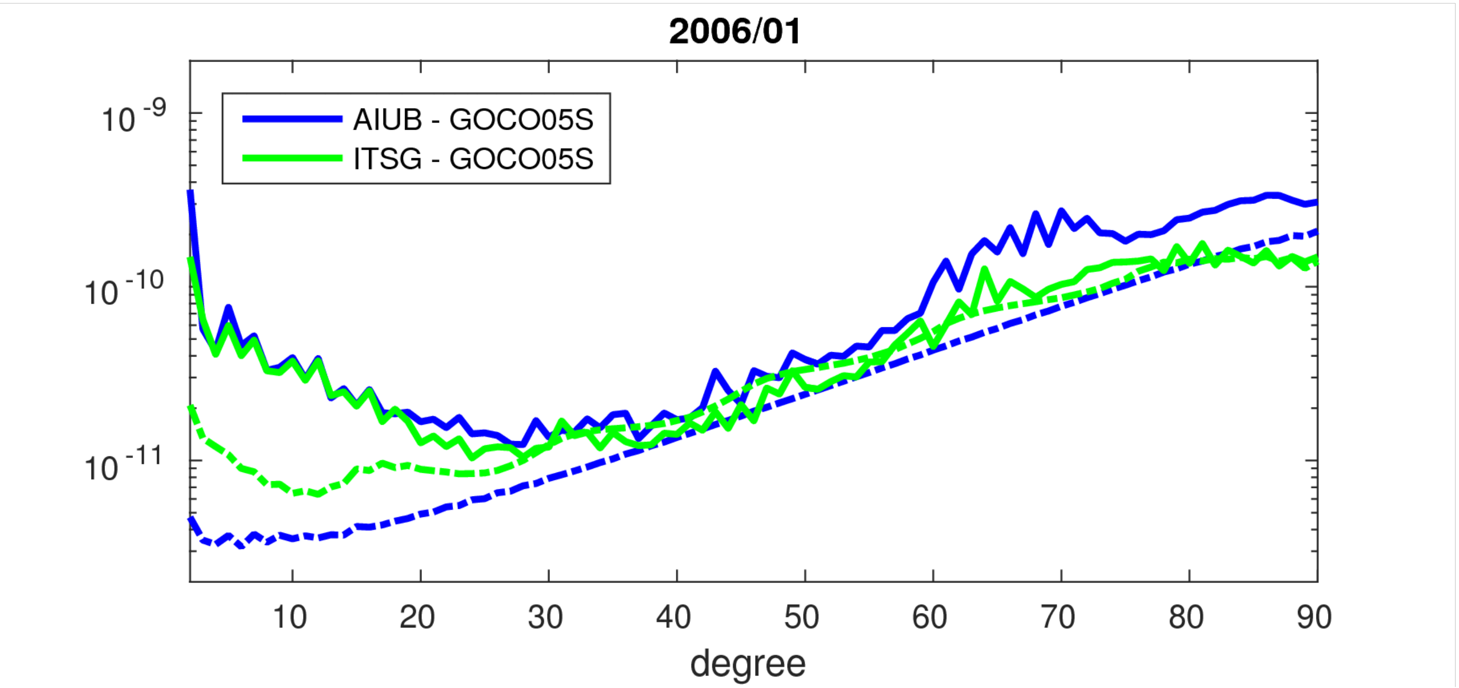
- Degree amplitudes of anomalies with respect to modeled secular and seasonal variations (based on ICGEM dataset).
- Only orders 0..29 are considered: evaluation of part of the spectrum that is determined meaningful.

Individual Contributions: AIUB



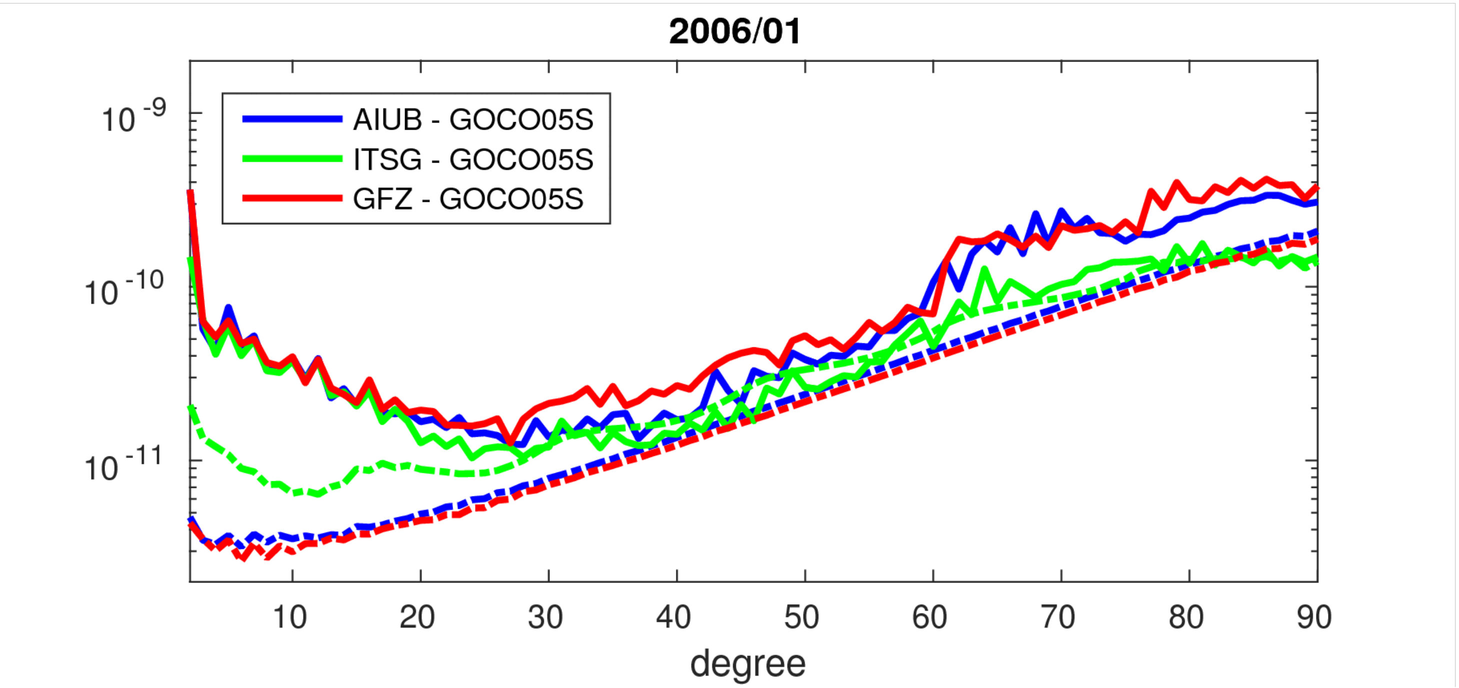
- **AIUB:** Celestial mechanics approach (dynamic approach relying on frequent pseudo-stochastic accelerations)
  - **approx. 500000 KRR observations and**
  - **500000 kinematic positions (30s) / month**

Individual Contributions: ITSG



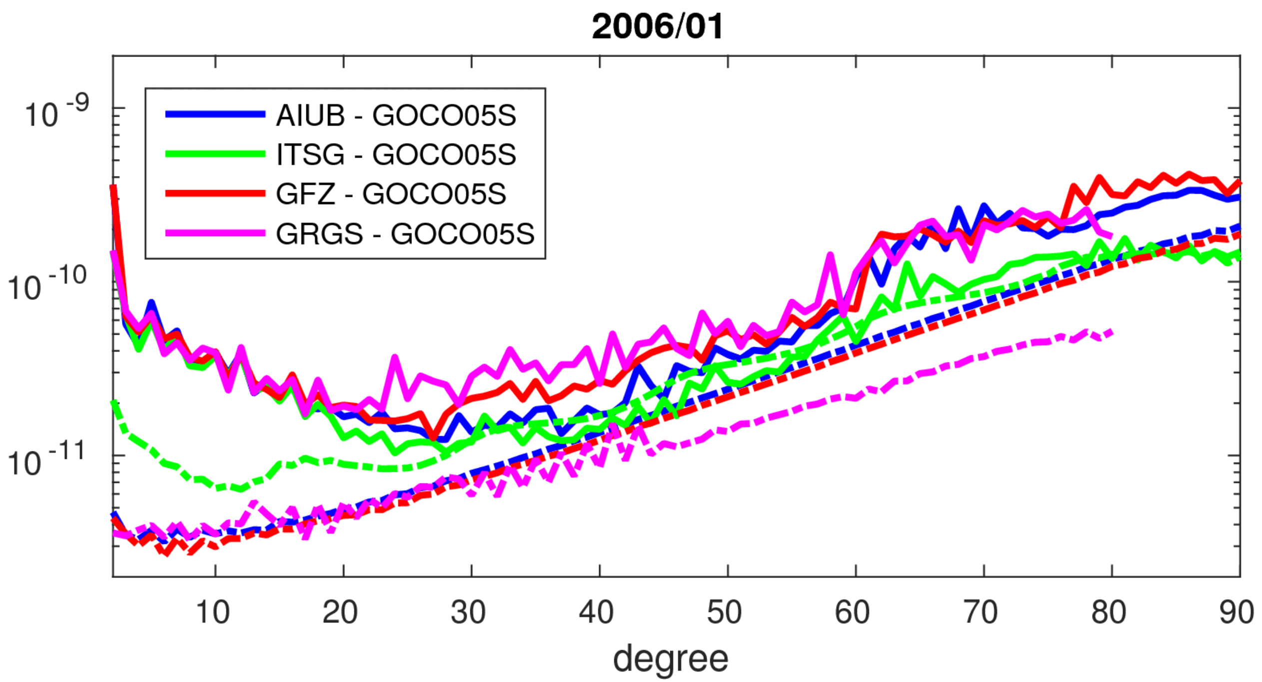
- **ITSG:** originally short arc approach, empirical noise model
  - **approx. 500000 KRR observations and**
  - **50000 kinematic positions (300s) / month**

Individual Contributions: GFZ



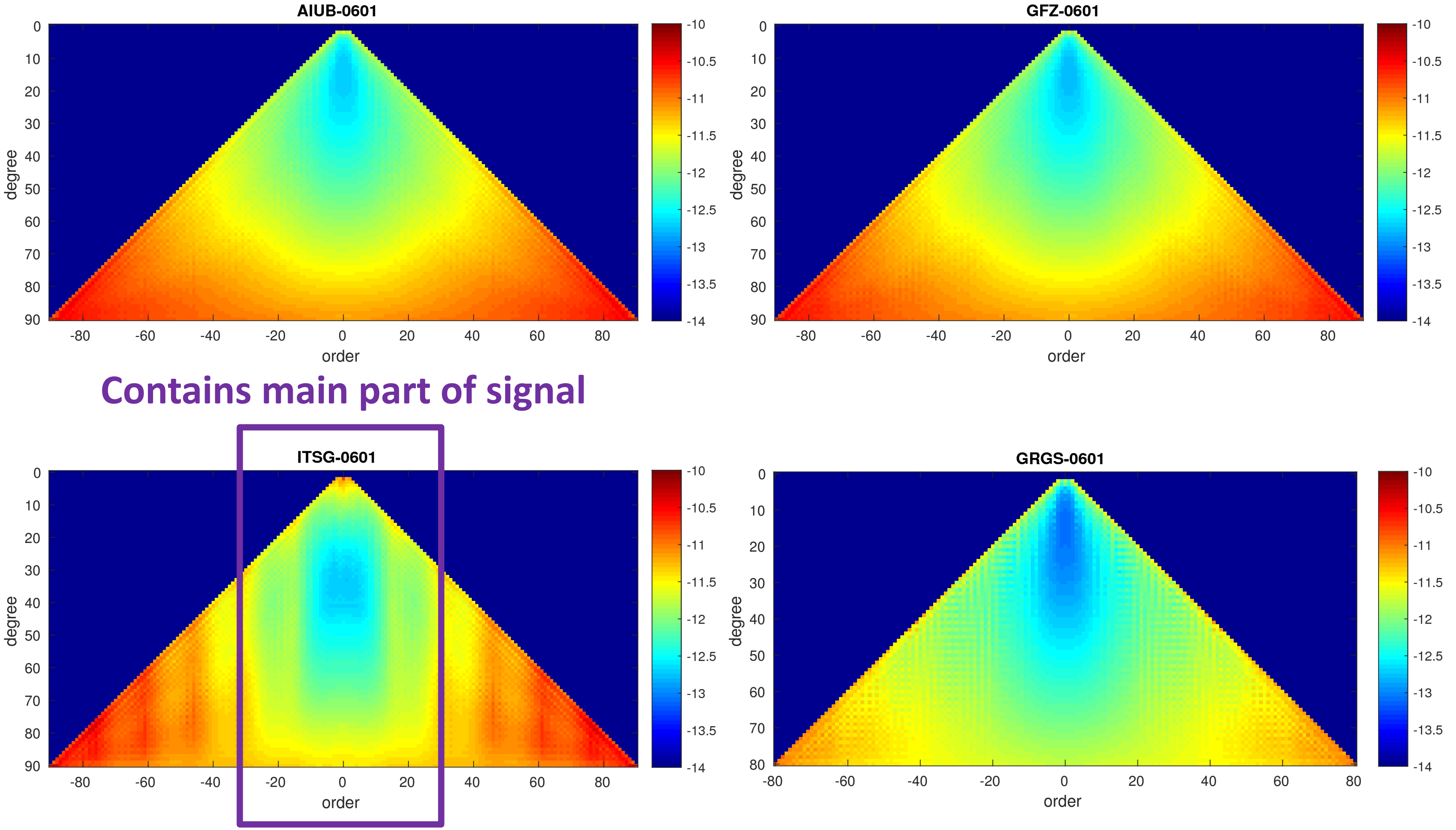
- **GFZ:** dynamic approach, dense accelerometer parametrization
  - **approx. 500000 KRR observations and**
  - **approx. 2500000 GPS observations / month**

Individual Contributions: GRGS



- **GRGS:** magic approach
  - **approx. 500000 KRR observations and**
  - **approx. 2500000 GPS observations / month**

Formal errors: 2006/01



Variance Component Estimation

Iterative determination of weights:

$$w_{i,0} = 1 / \sigma_{i,0}^2 \; ; \; \sigma_{i,0}^2 = 1$$

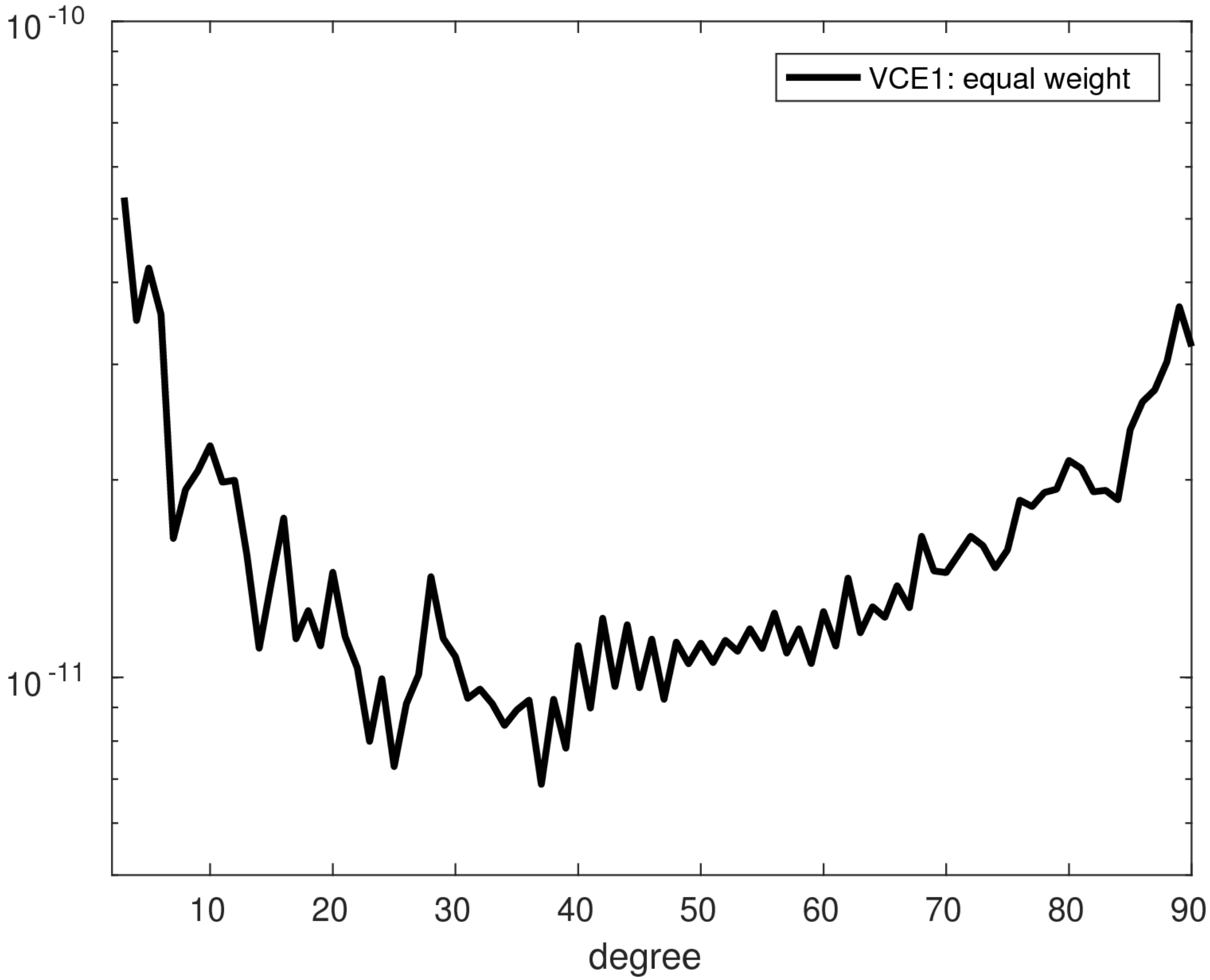
$$(\textstyle\sum_i w_{i,k}\, \mathbf{N}_i)\, \mathbf{dx} = \textstyle\sum_i w_{i,k}\, \mathbf{b}_i \;\; ; \;\; \mathbf{l}_{i,k}^T \mathbf{P}_{i,k} \mathbf{l}_{i,k} = w_{i,k}\, \mathbf{l}_i^T \mathbf{P}_i \mathbf{l}_i$$

$$\sigma_{i,k+1}^2 = \mathbf{v}_{i,k}^T\, \mathbf{P}_i\, \mathbf{v}_{i,k} / r_i$$

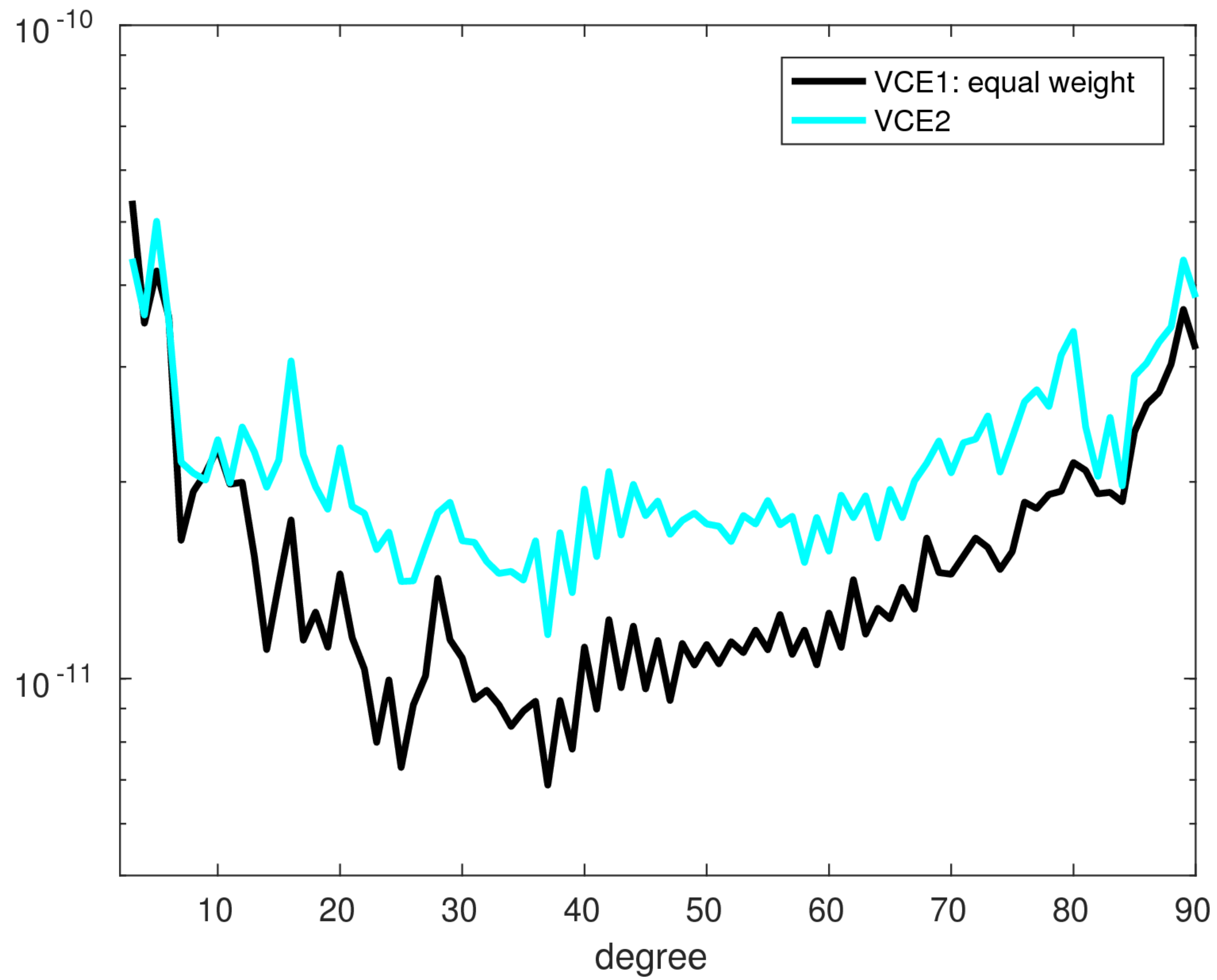
Square sum of residuals: $\mathbf{v}_{i,k}^T\, \mathbf{P}_i\, \mathbf{v}_{i,k} = \mathbf{l}_i^T\, \mathbf{P}_i\, \mathbf{l}_i - \mathbf{b}_i^T\, \mathbf{dx}_k$

Partial redundancy: $r_i = n_i - m$

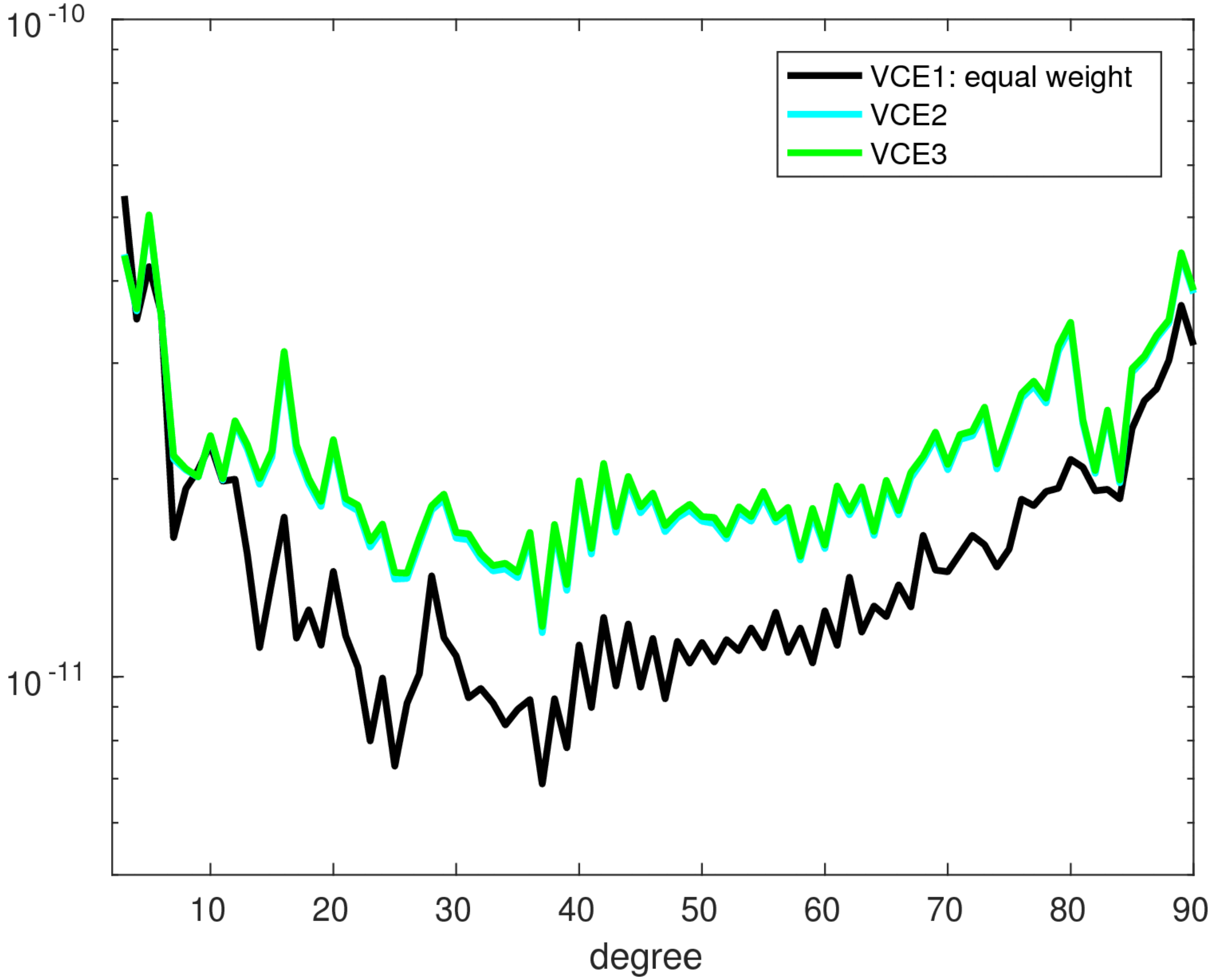
Variance Component Estimation (0)



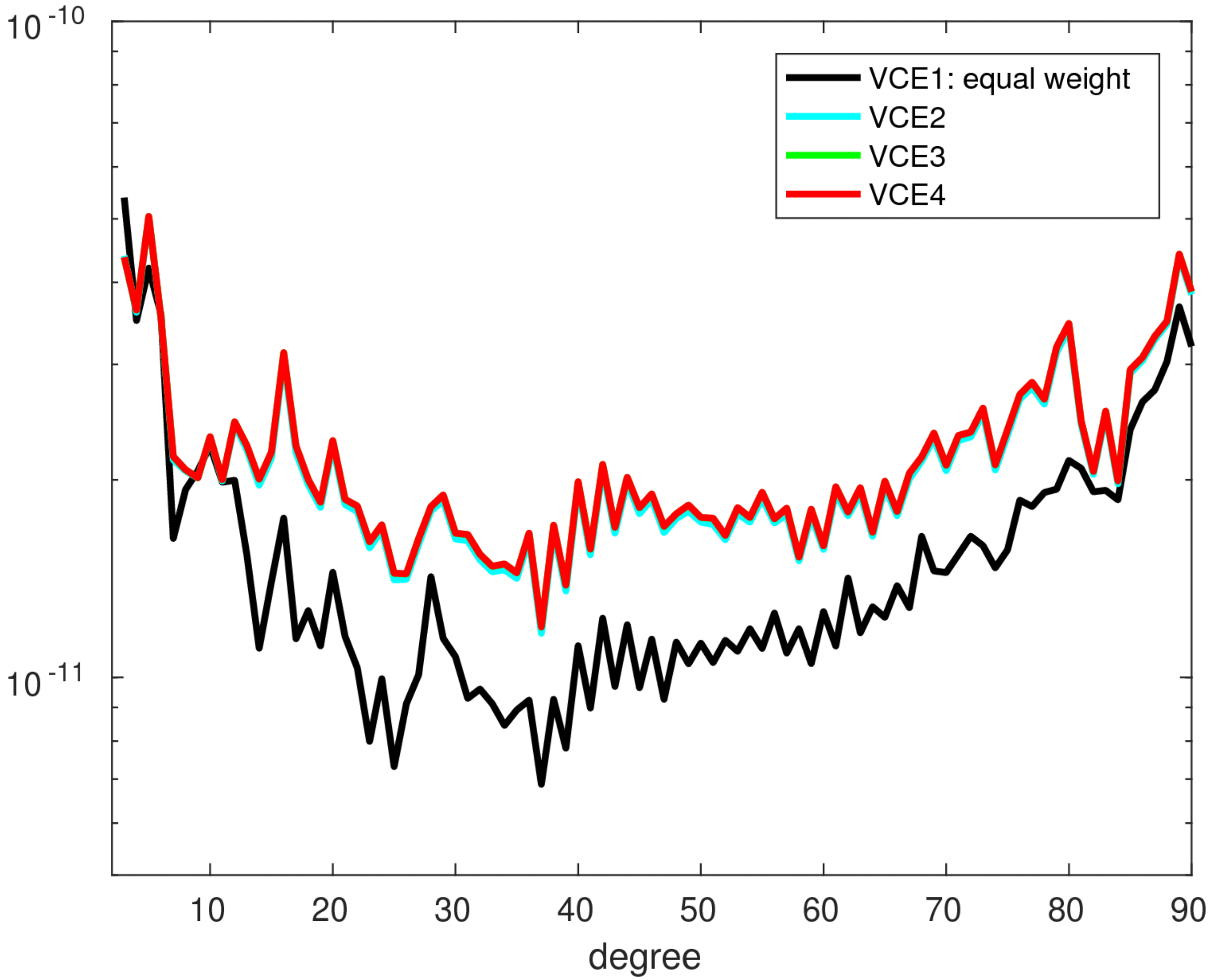
Variance Component Estimation (1)



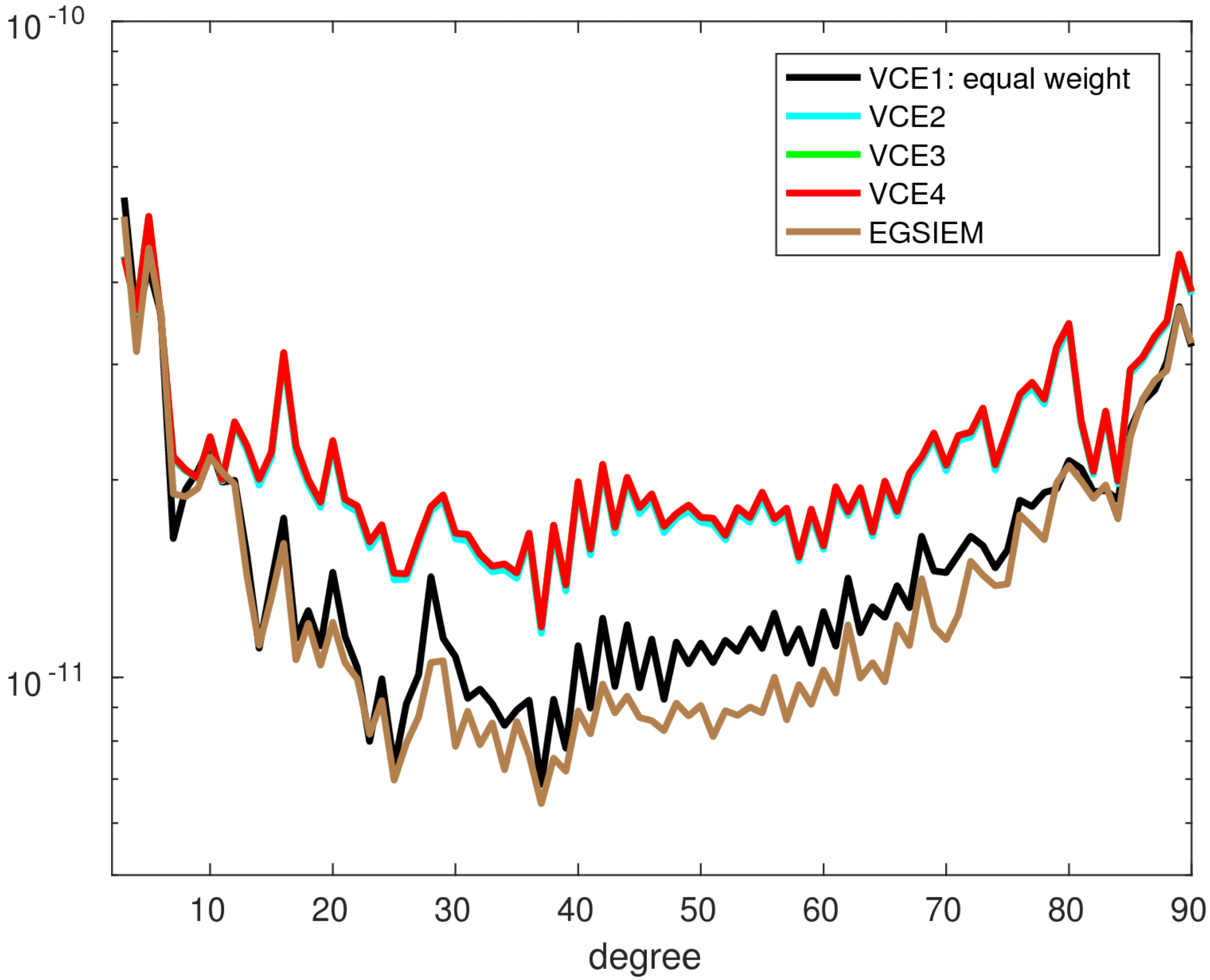
Variance Component Estimation (2)



Variance Component Estimation (3)

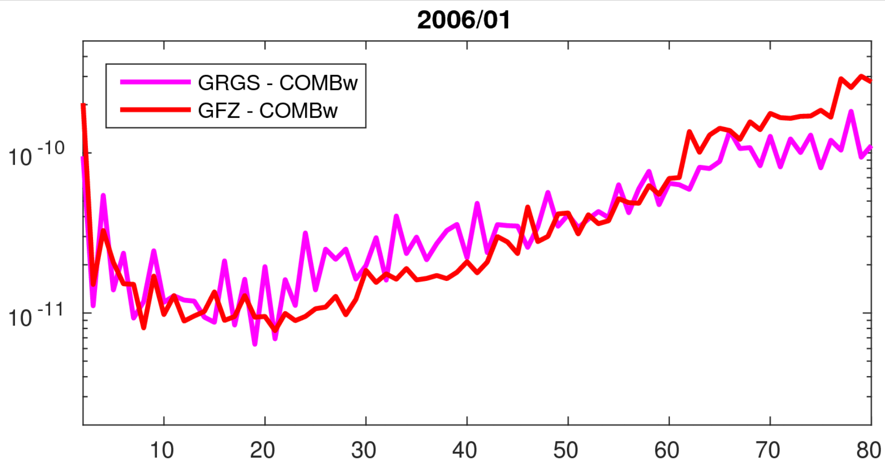


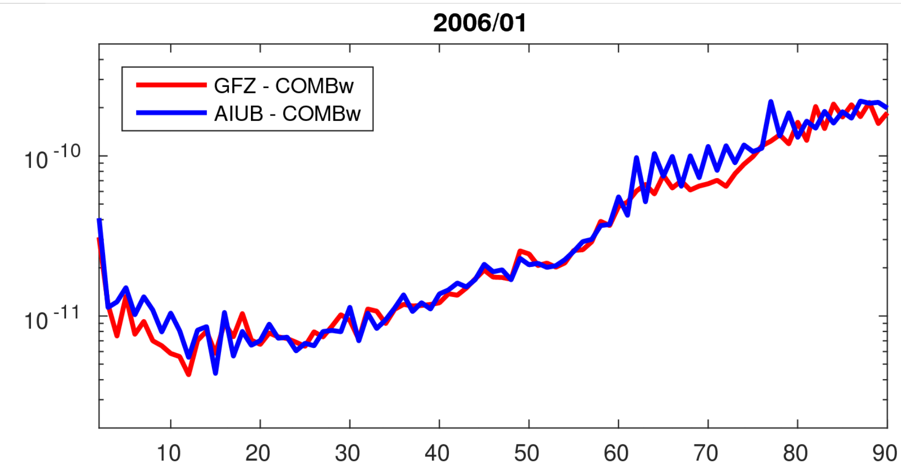
Variance Component Estimation (4)

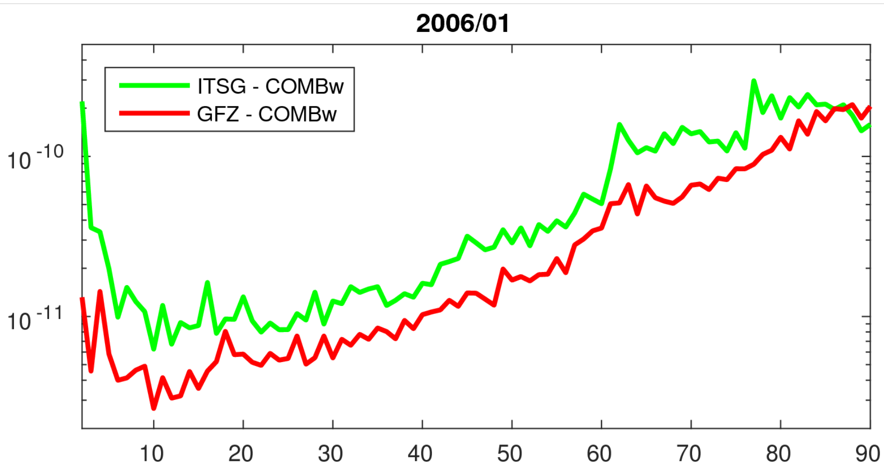


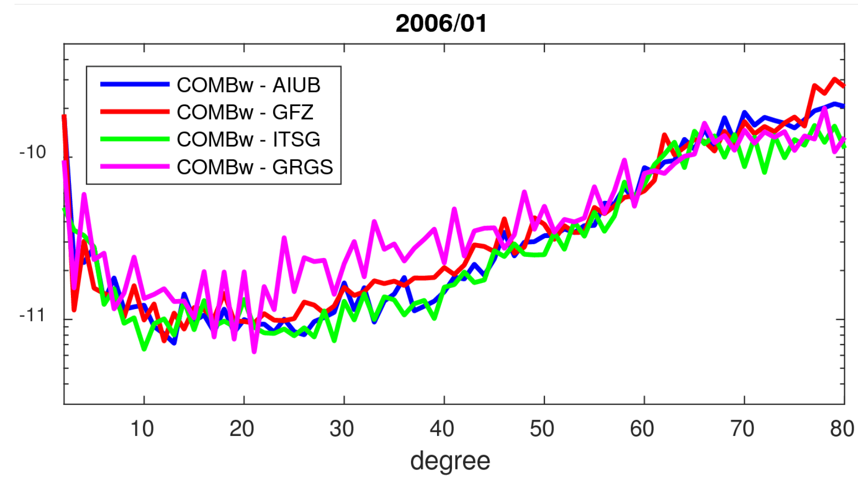
VCE	weight
GRGS	3.23
GFZ	0.87
AIUB	5.88
ITSG	1.08

Individual contributions (variance factors): 2006/01









Empirical rescaling to achieve equal impact

A straight-forward empirical approach is to search for weights w_i that equalize the impact of individual contributions on pairwise combinations:

$$(\mathbf{N}_{ref} + w_i\,\mathbf{N}_i)\,\mathbf{dx} = \mathbf{b}_{ref} + w_i\,\mathbf{b}_i$$

The impact is measured by:

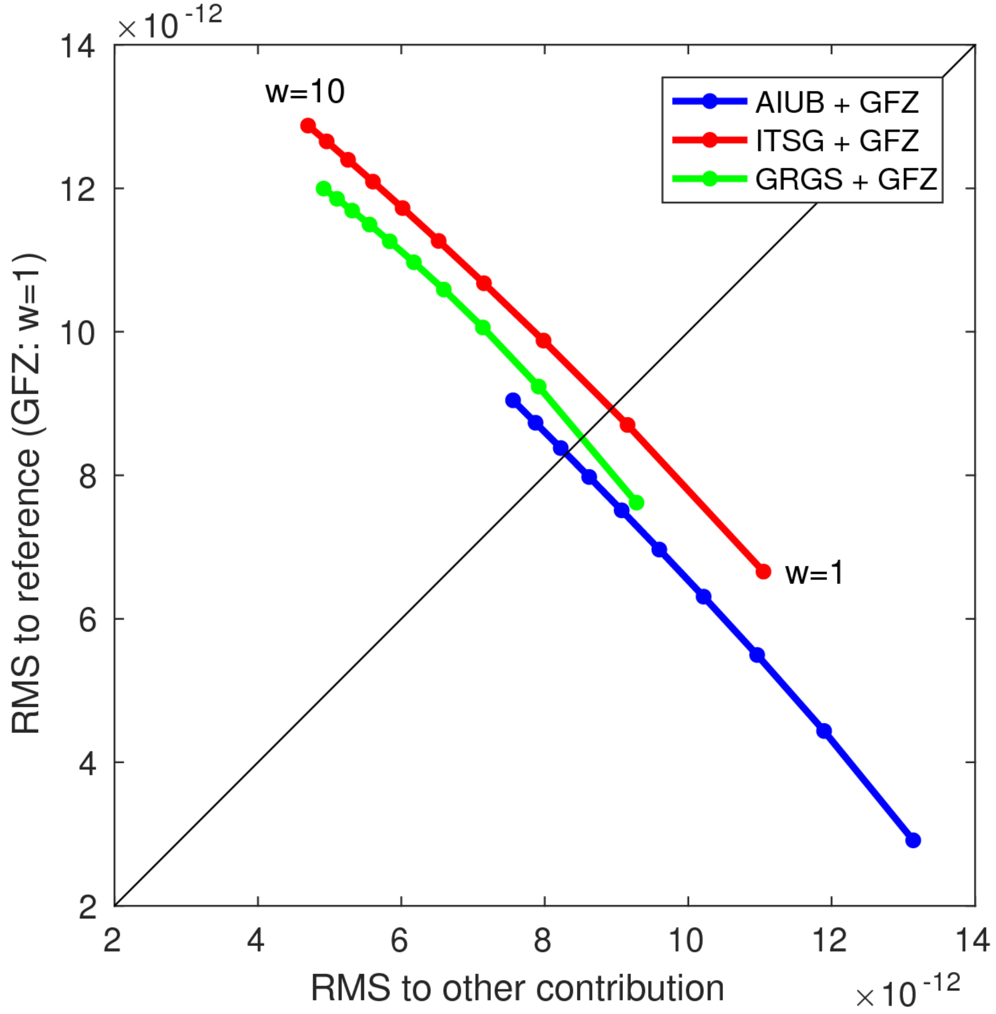
$$RMS_i = SQRT(\textstyle\sum_{l,m}(K_{l,m}^{comb} - K_{l,m}^{i})^2/n_{coef})$$

Equal impact is achieve for:

$$RMS_i/RMS_{ref} = 1$$

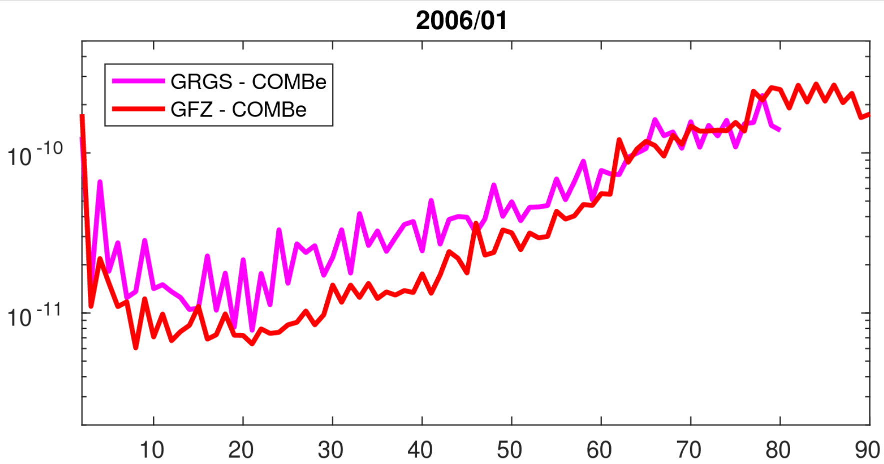
Consequently weights derived on solution level are applied.

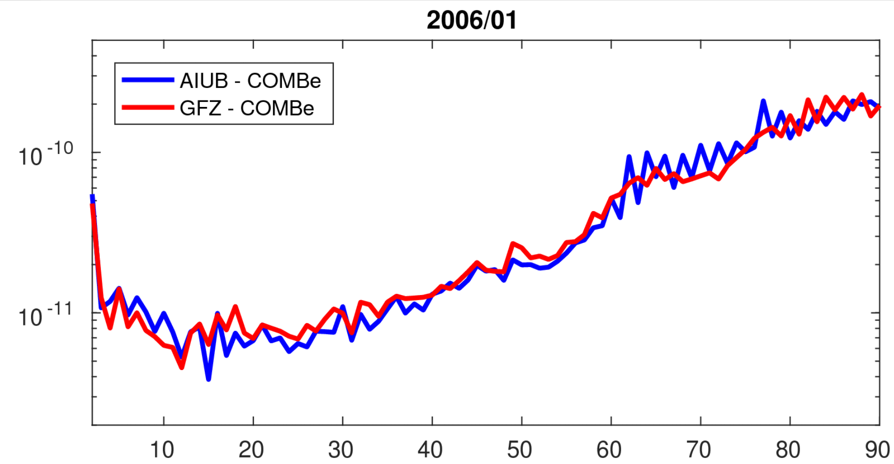
Empirical rescaling to achieve equal impact

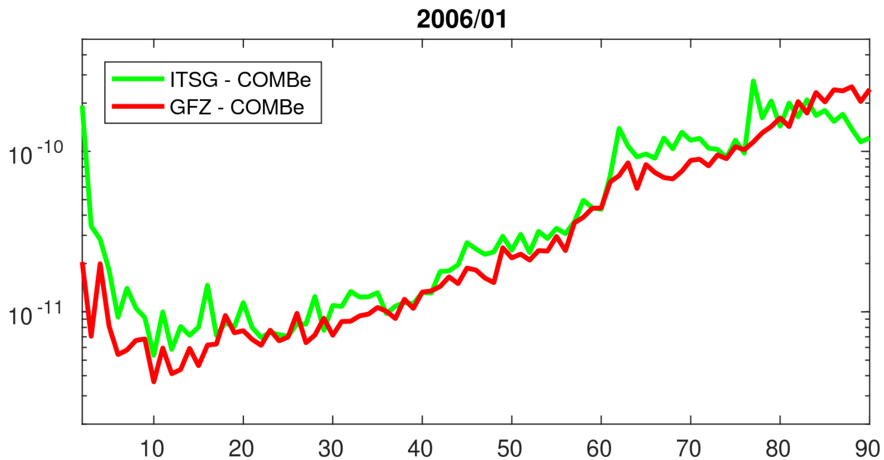


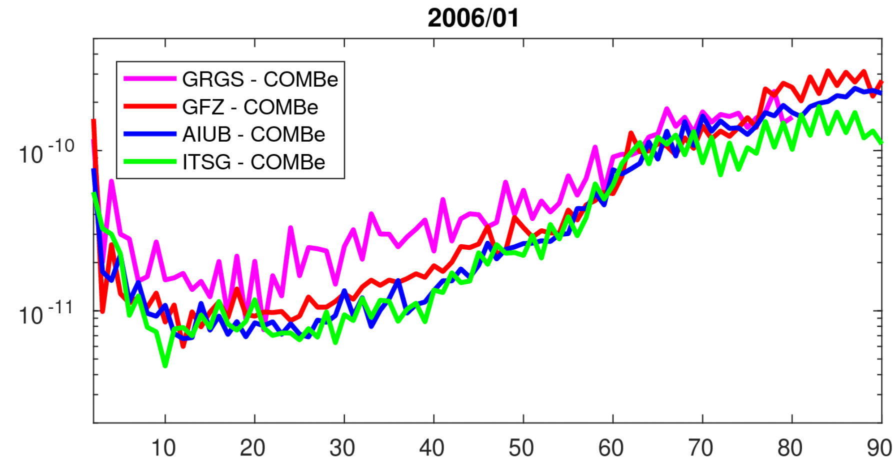
equalizing weight	
GRGS	1.60
GFZ	1.00
AIUB	7.81
ITSG	2.21

Individual contributions (equalized): 2006/01

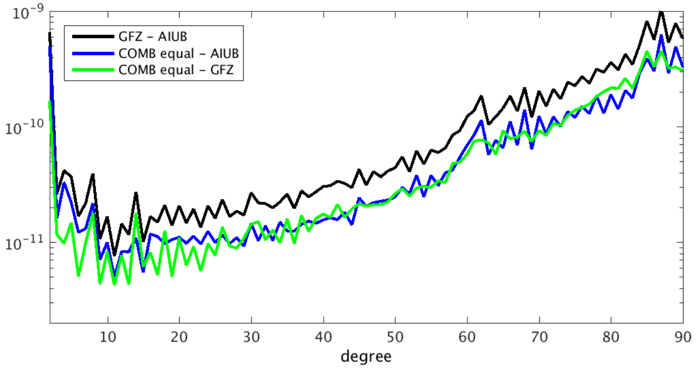


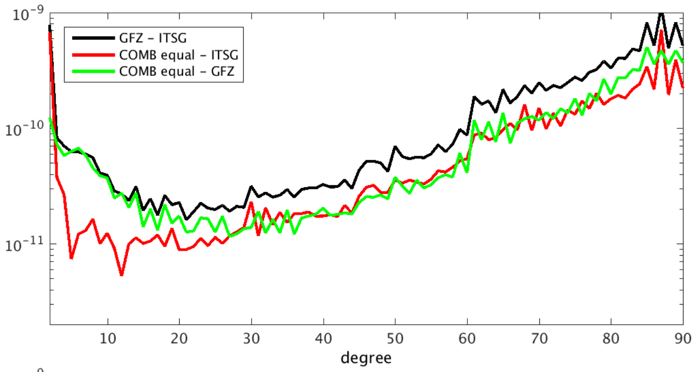


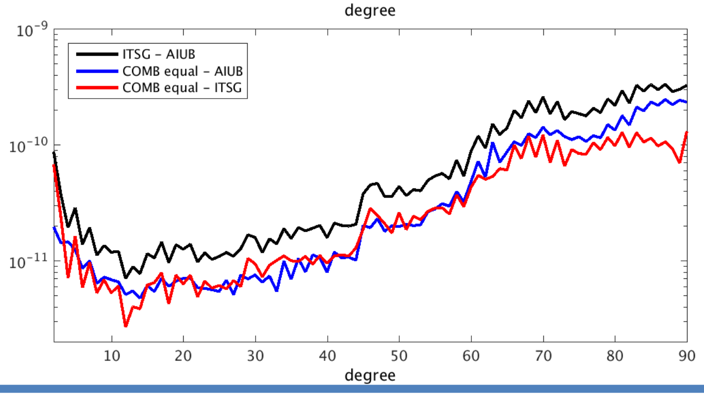


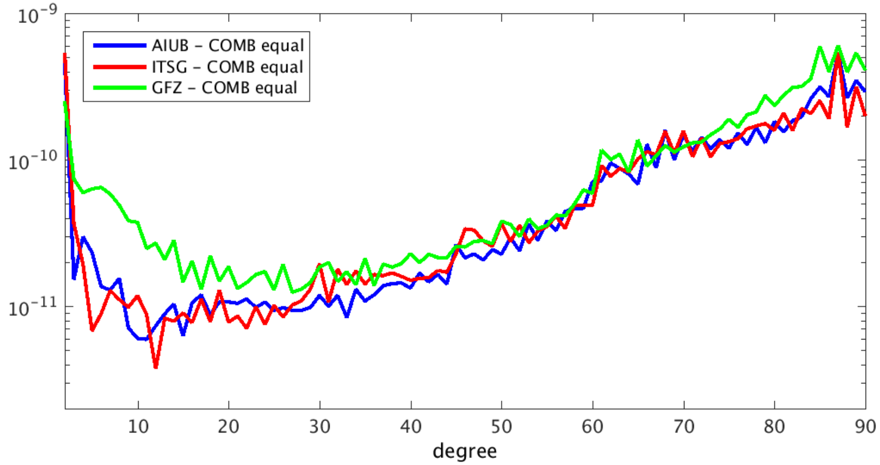


# Equal contribution by empirical weighting

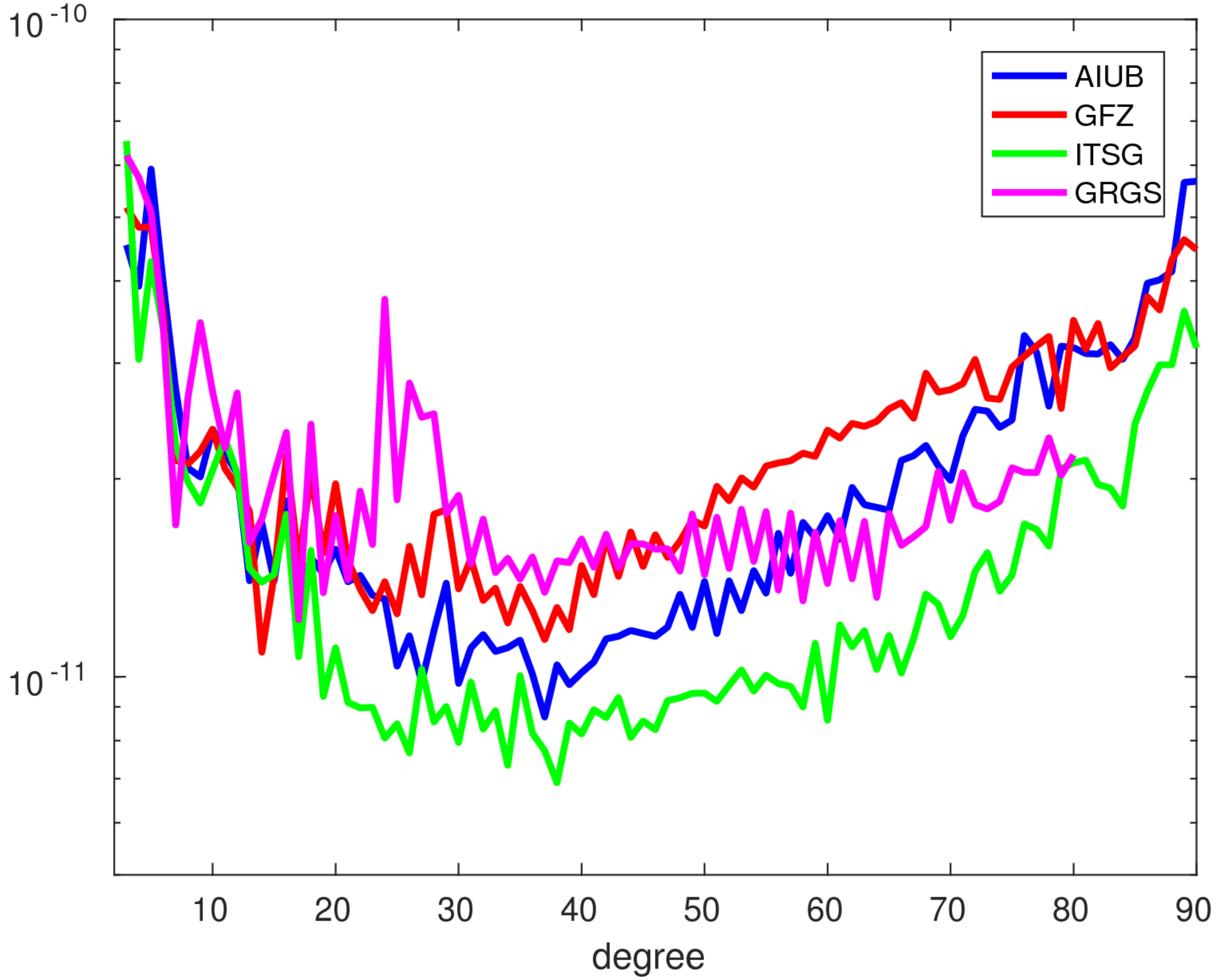




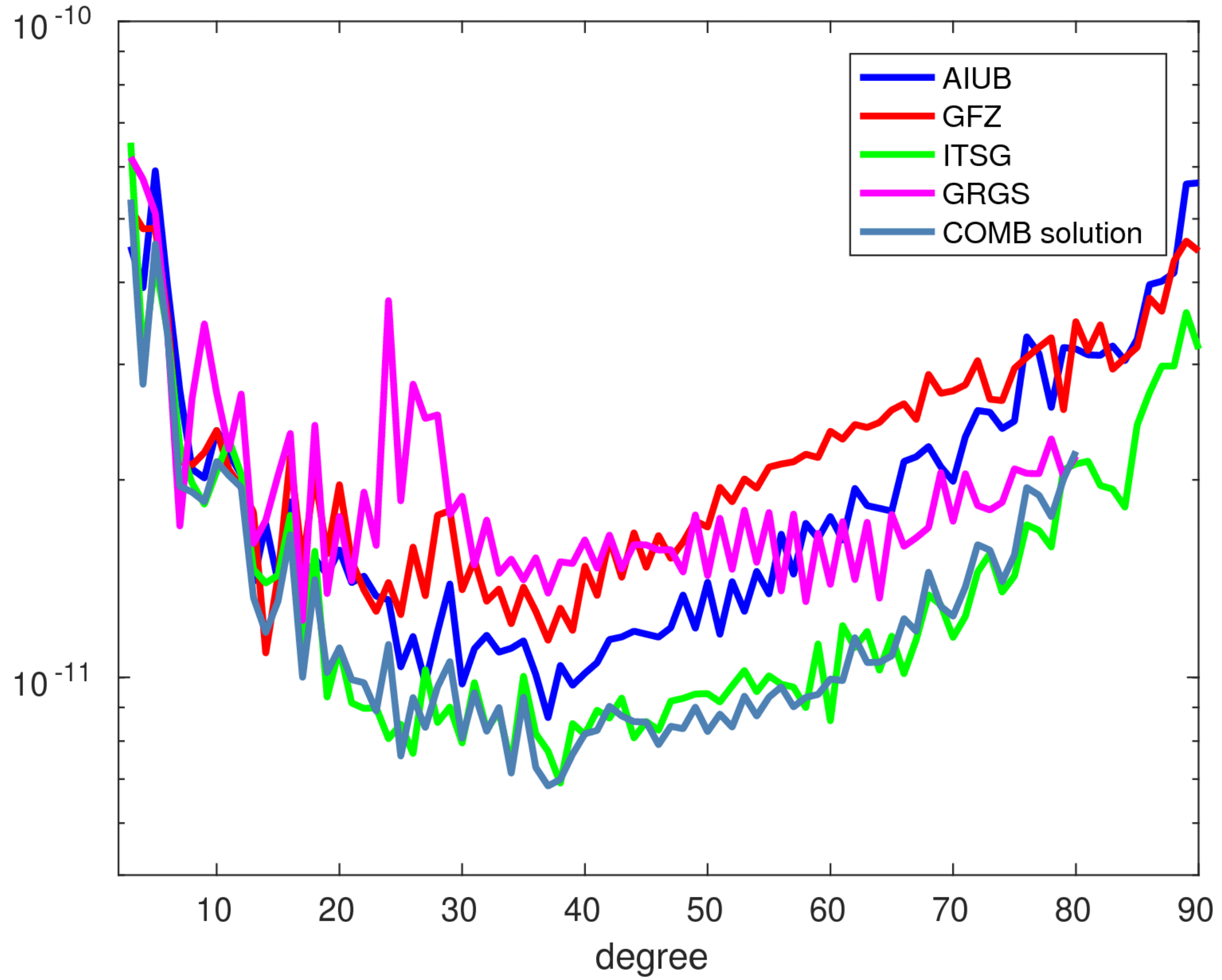




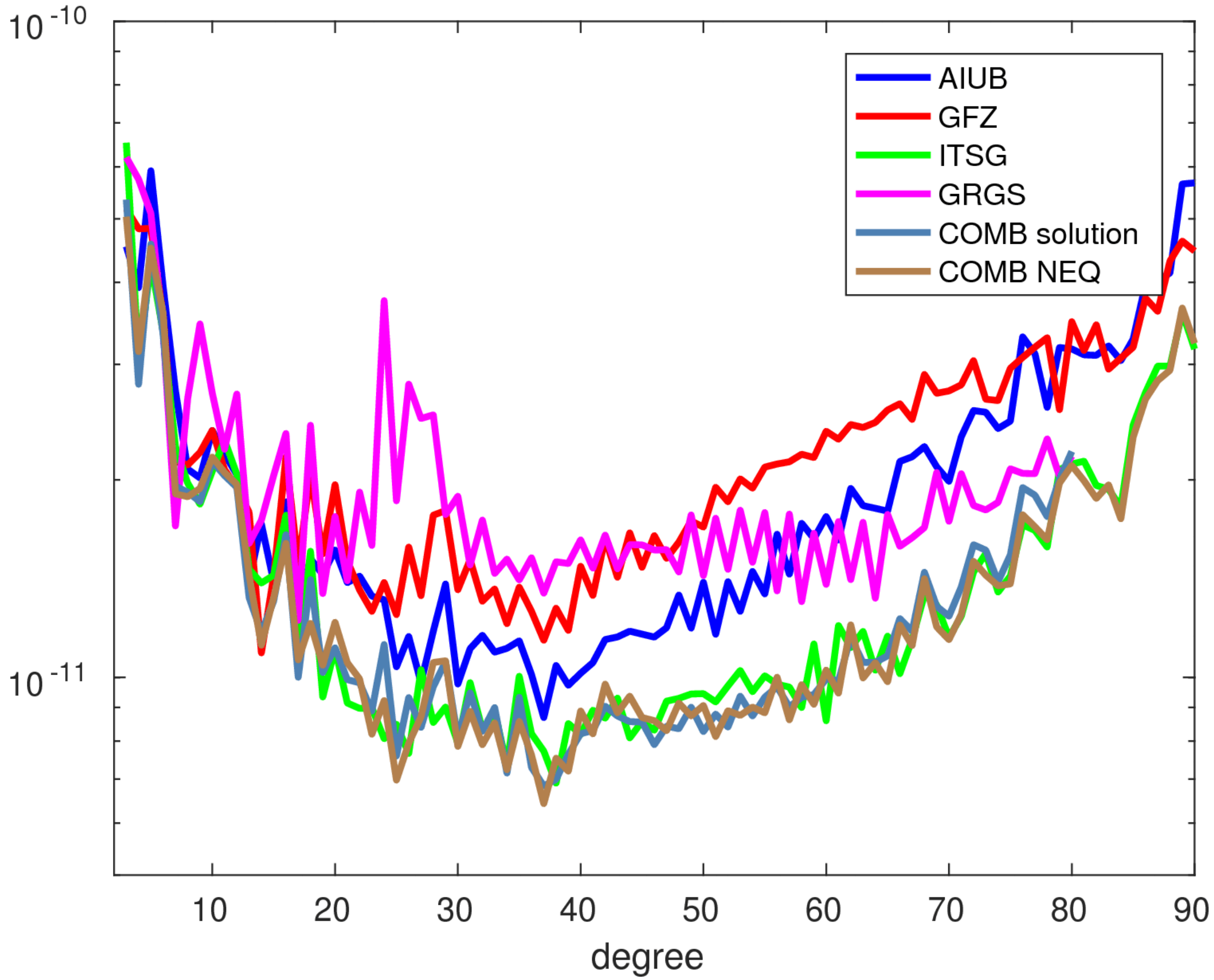
Individual Solutions 2006/01



Weighted Combination on Solution Level



Weighted Combination on NEQ-level



equalizing weight	
GRGS	1.60
GFZ	1.00
AIUB	7.81
ITSG	2.21

Solution:	weight
GRGS	0.14
GFZ	0.19
AIUB	0.29
ITSG	0.38

Weighting schemes: comparison

equal

GRGS	0.25
GFZ	0.25
AIUB	0.25
ITSG	0.25

VCE

GRGS	0.29
GFZ	0.08
AIUB	0.53
ITSG	0.10

=

equalizing

GRGS	0.13
GFZ	0.08
AIUB	0.62
ITSG	0.17

* **VCE**

GRGS	0.49
GFZ	0.21
AIUB	0.18
ITSG	0.12

=

GRGS	0.29
GFZ	0.08
AIUB	0.53
ITSG	0.10

* **solution level**

GRGS	0.14
GFZ	0.19
AIUB	0.29
ITSG	0.38

=

GRGS	0.07
GFZ	0.05
AIUB	0.65
ITSG	0.23

Combination on Normal Equation Level

What can we do to a normal equation without changing the individual solution:

$$\mathbf{N\,dx = b\ ;\ x = x_0 + dx}$$

Scalar scaling: $\quad f\,\mathbf{N\,dx} = f\,\mathbf{b}$

Matrix scaling: $\quad \mathbf{F^T\,N\,F\,F^{-1}\,dx = F^T\,b\ ;\ x_0' = F^{-1}\,x_0}$

Transformation to different a priori values:

$$\mathbf{x_0' = x_0 + dx_0\ ;\ N\,(dx - dx_0) = b - N\,dx_0}$$

Rescaling of formal errors

Cofactor matrix: $\mathbf{Q}' = \mathbf{S}\ \mathbf{Q}\ \mathbf{S}$; $s_{ii} = \sigma_{ii} / \sigma_{ii,ref}$; $s_{ij} = 0$

Normal matrix: $\mathbf{F}^T\ \mathbf{N}\ \mathbf{F} = (\mathbf{S}\ \mathbf{Q}\ \mathbf{S})^{-1}$

Cholesky decomposition: $\mathbf{N} = \mathbf{G}\ \mathbf{G}^T$

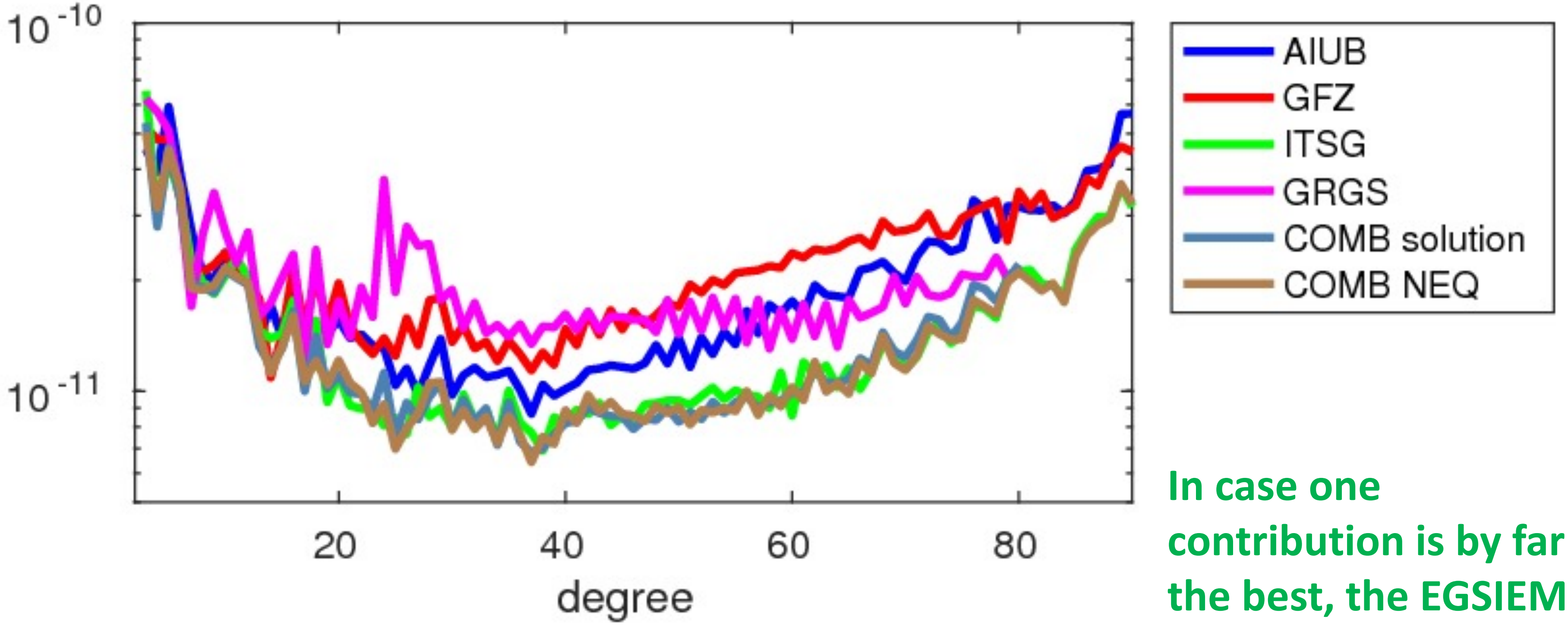
$(\mathbf{S}\ \mathbf{Q}\ \mathbf{S})^{-1} = \mathbf{H}\ \mathbf{H}^T$

$$\mathbf{F}^T\ \mathbf{G}\ \mathbf{G}^T\ \mathbf{F} = \mathbf{H}\ \mathbf{H}^T \quad \Rightarrow \quad \mathbf{F}^T = \mathbf{H}\ \mathbf{G}^{-1}$$

Resulting NEQ: $\mathbf{N}'\ \mathbf{dx}' = \mathbf{b}'$

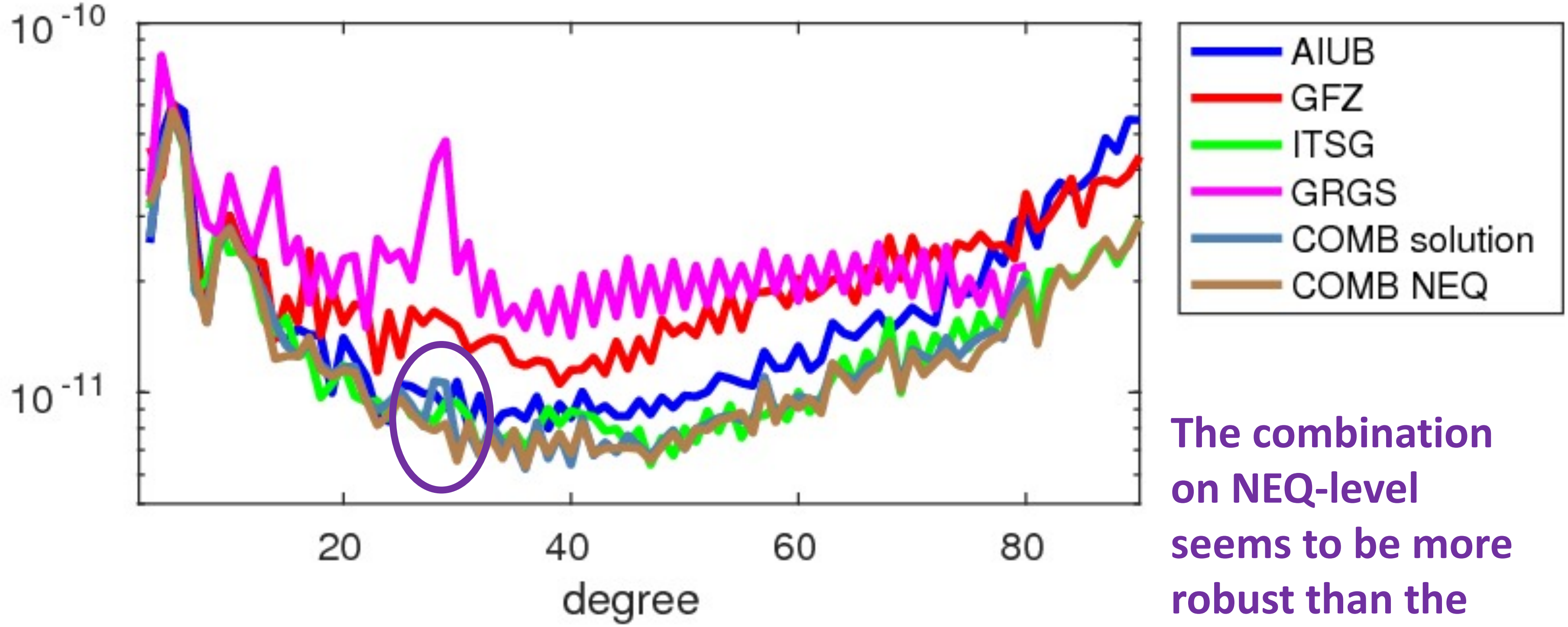
with $\mathbf{N}' = \mathbf{F}^T\ \mathbf{N}\ \mathbf{F}$, $\mathbf{b}' = \mathbf{F}^T\mathbf{b}$, $\mathbf{dx}' = \mathbf{F}^{-1}\mathbf{dx}$ and $\mathbf{x_0}' = \mathbf{F}^{-1}\ \mathbf{x_0}$

2006/01



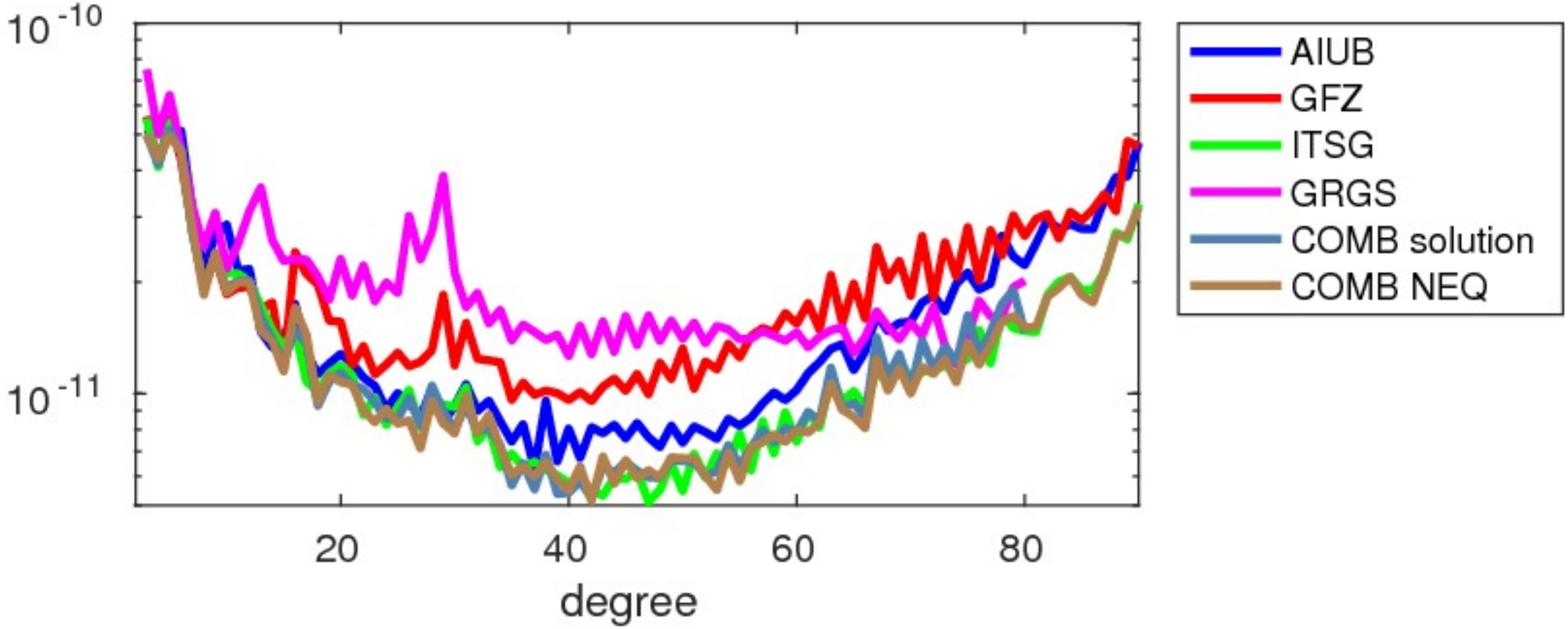
In case one contribution is by far the best, the EGSIEM-combinations are close to it.

2006/02



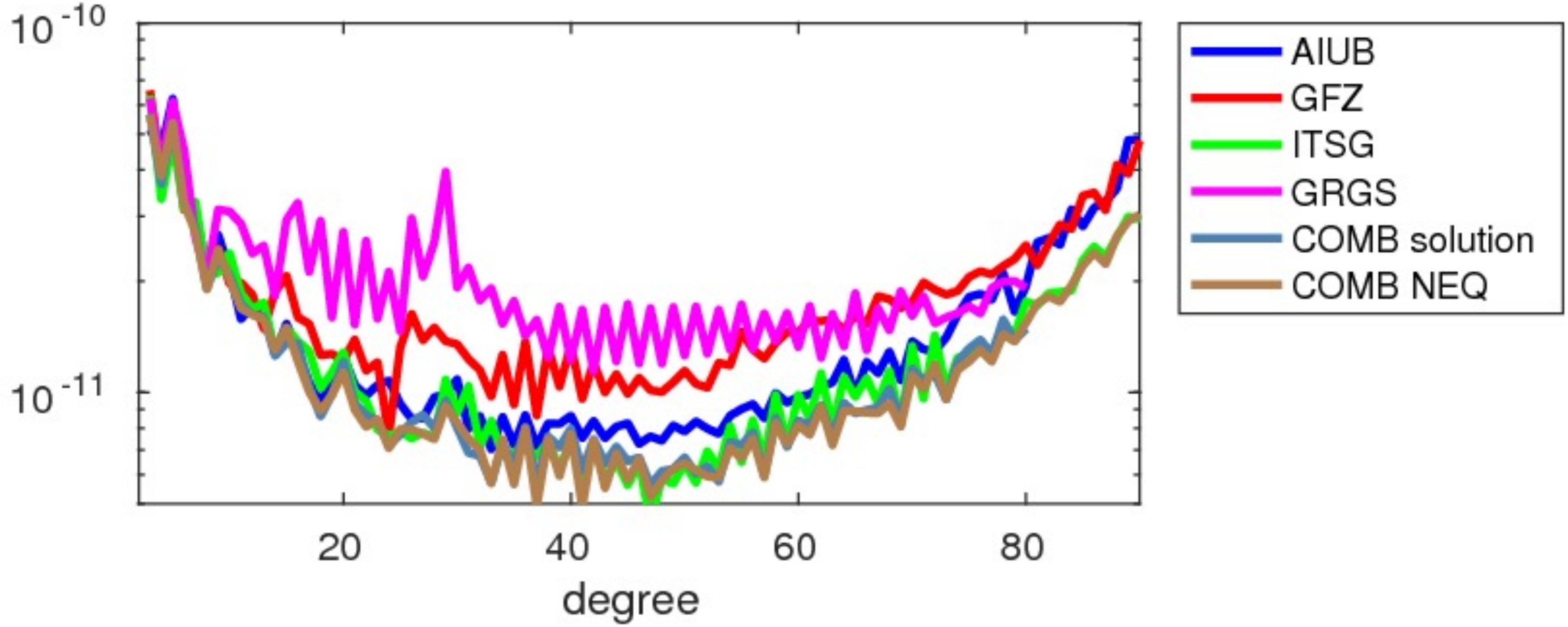
The combination on NEQ-level seems to be more robust than the combination on solution level.

2006/03



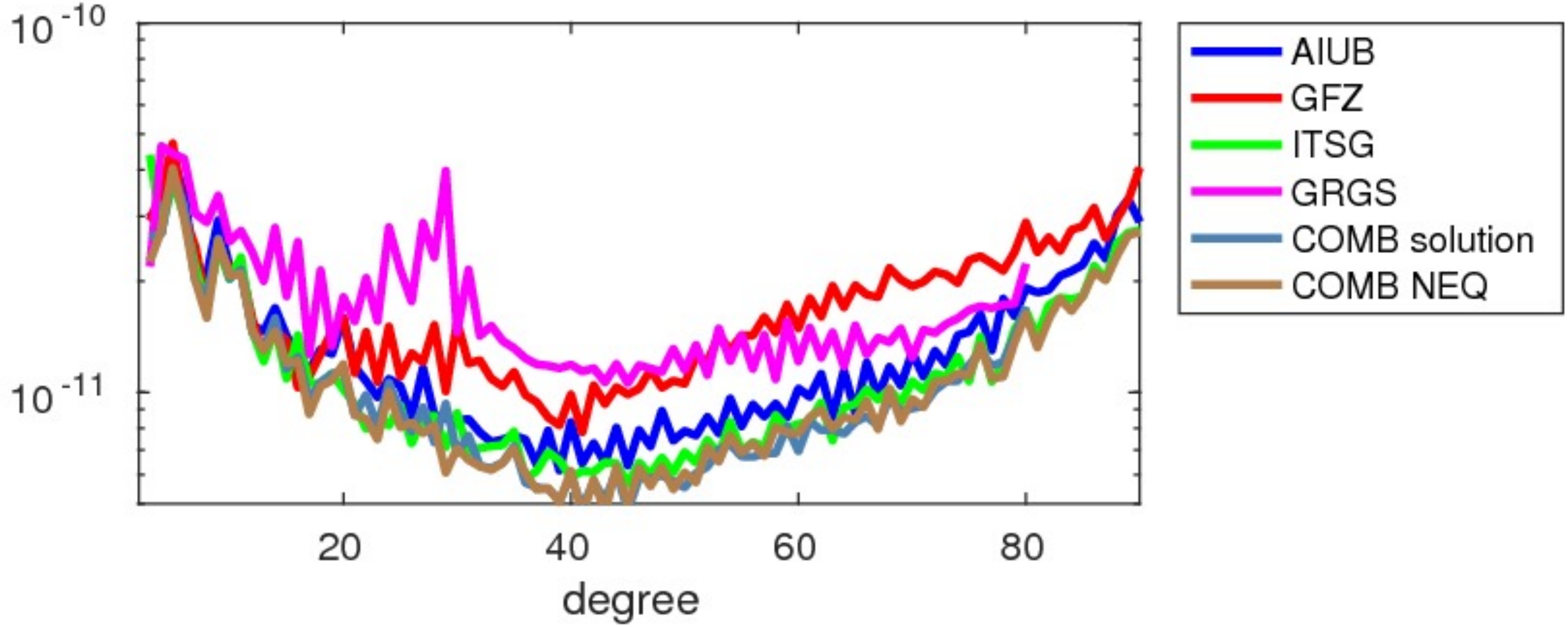
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2006/04



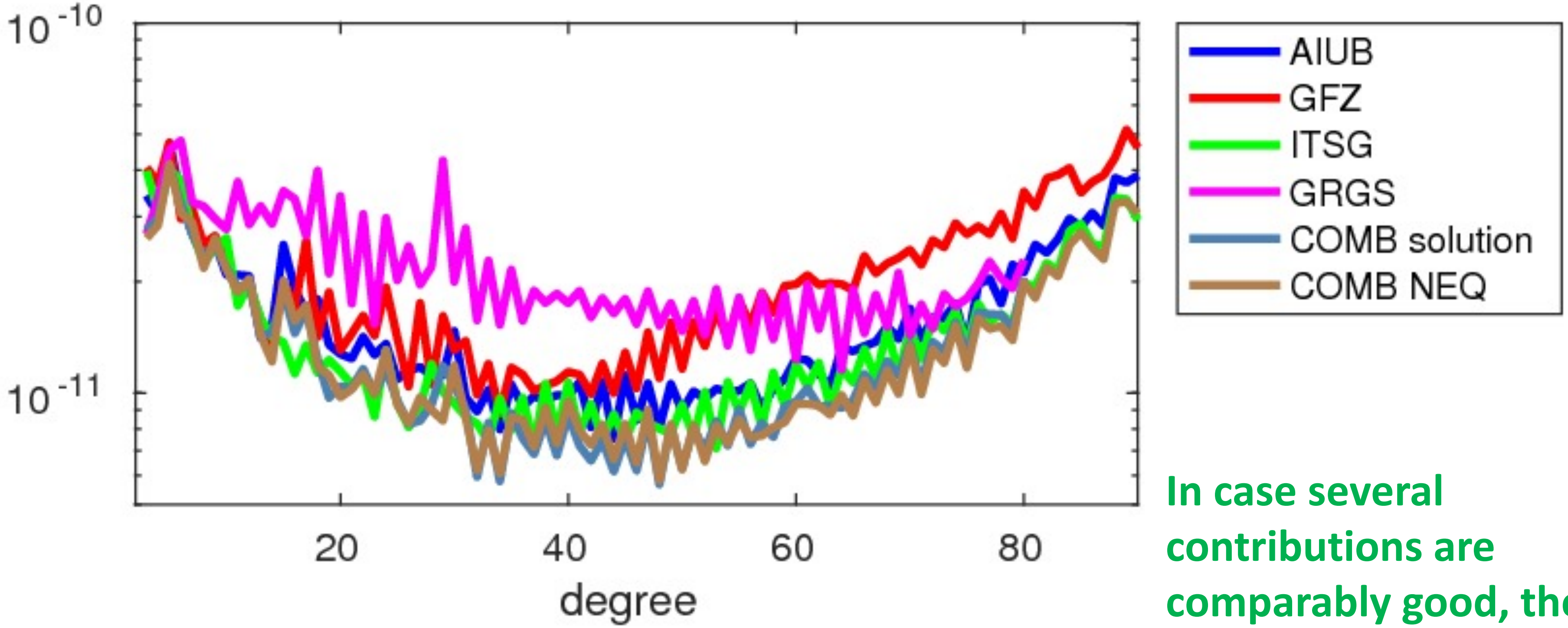
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2006/05



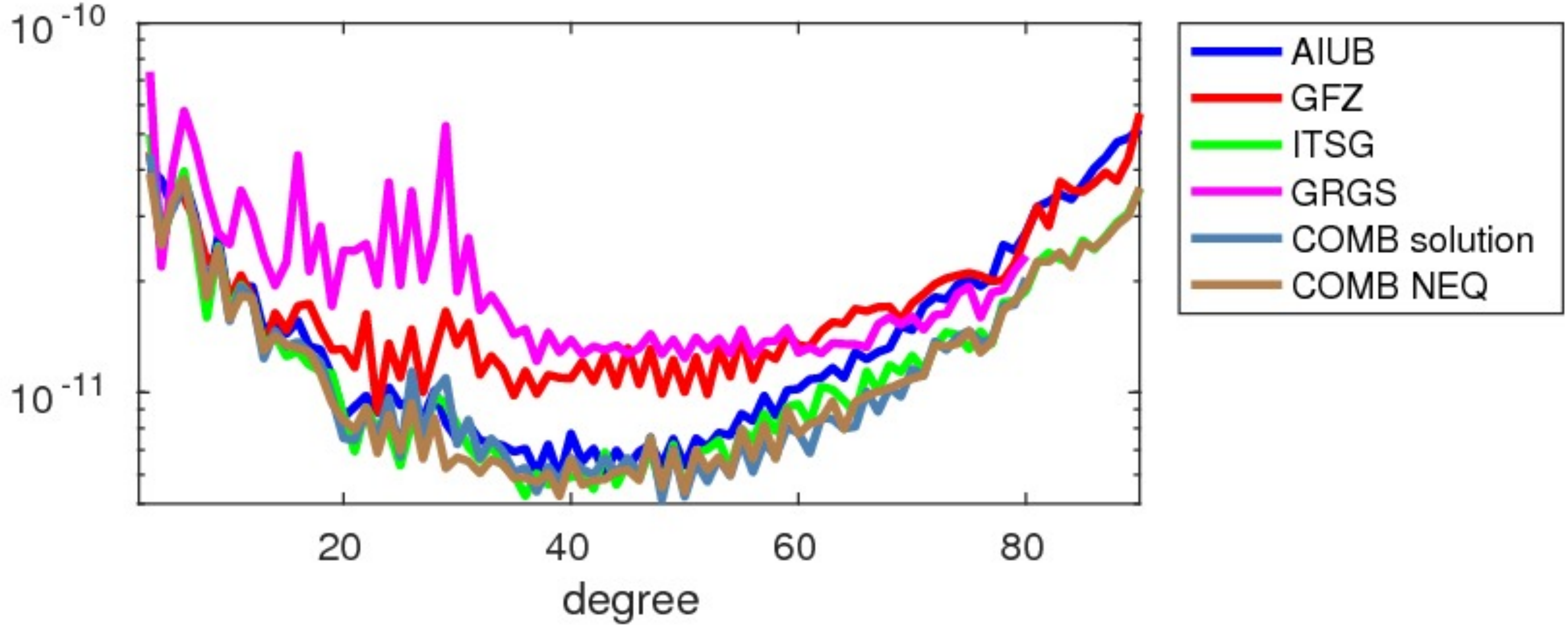
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2006/06



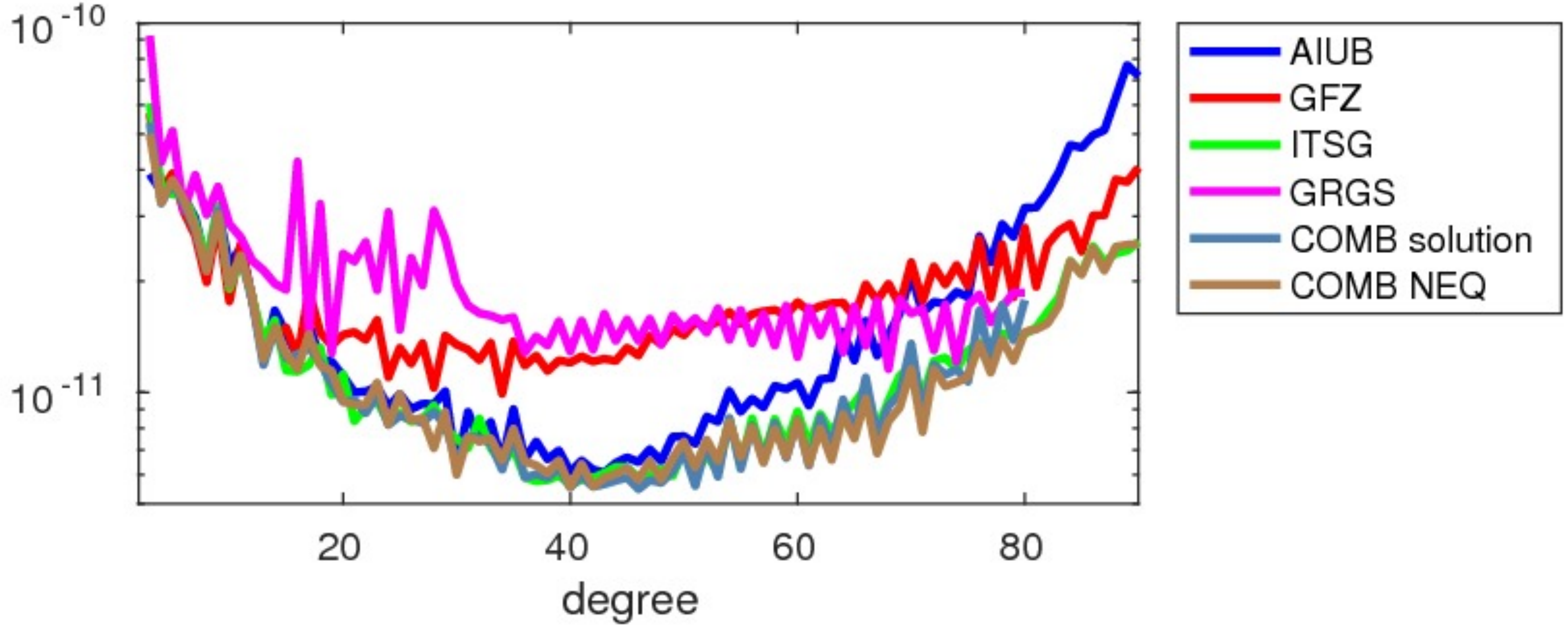
In case several contributions are comparably good, the EGSIEM-combinations are better!

2006/07



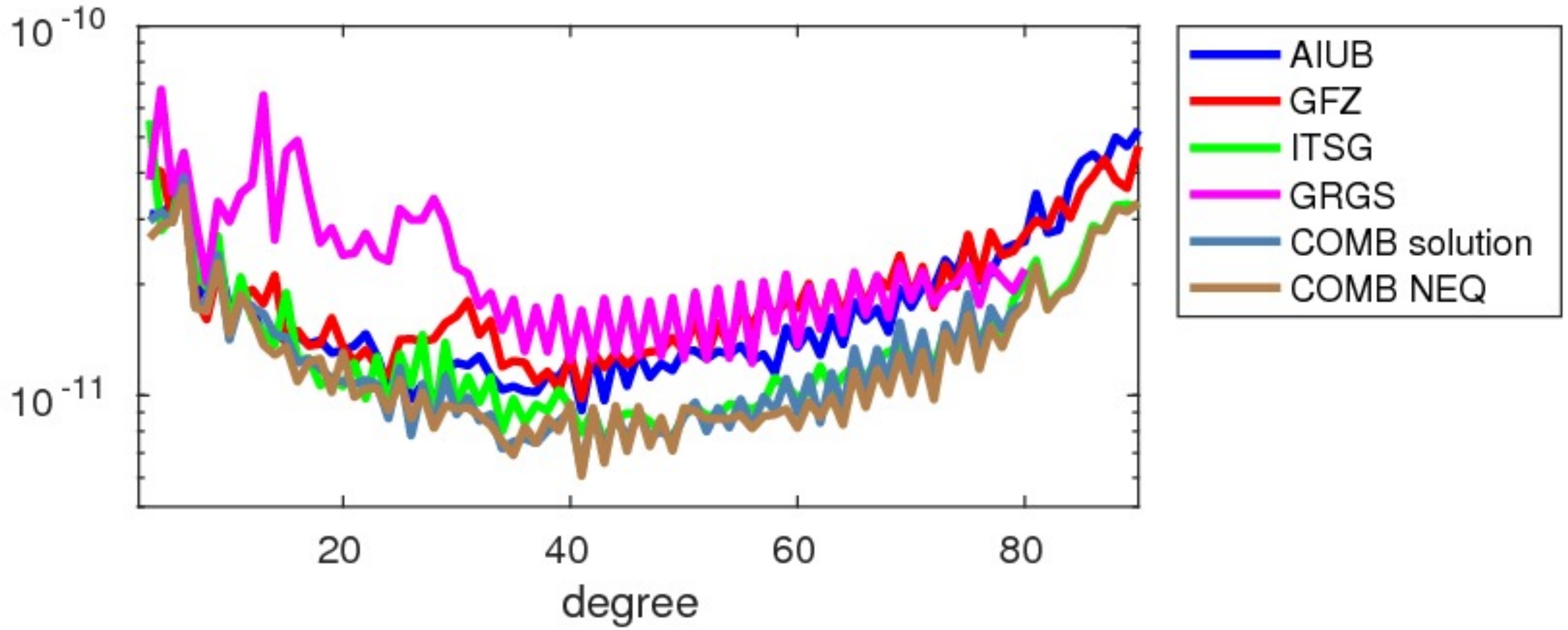
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2006/08



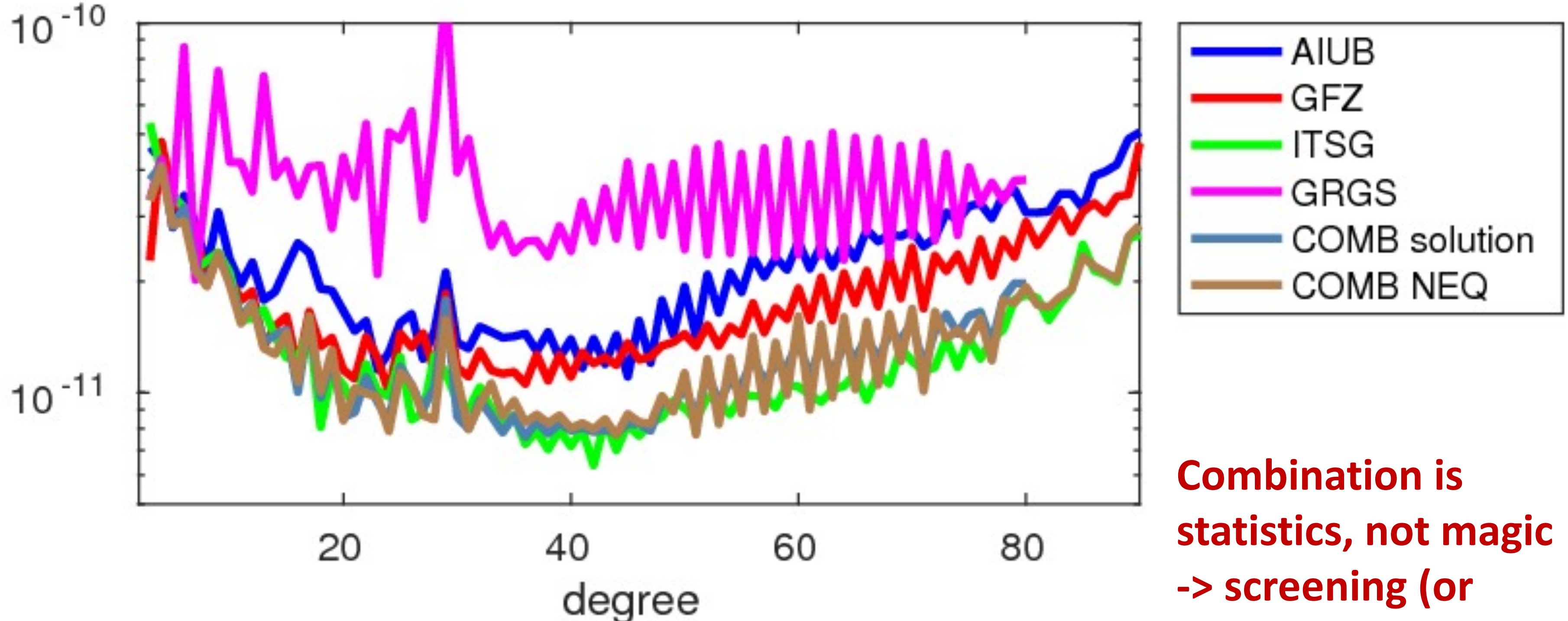
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2006/09



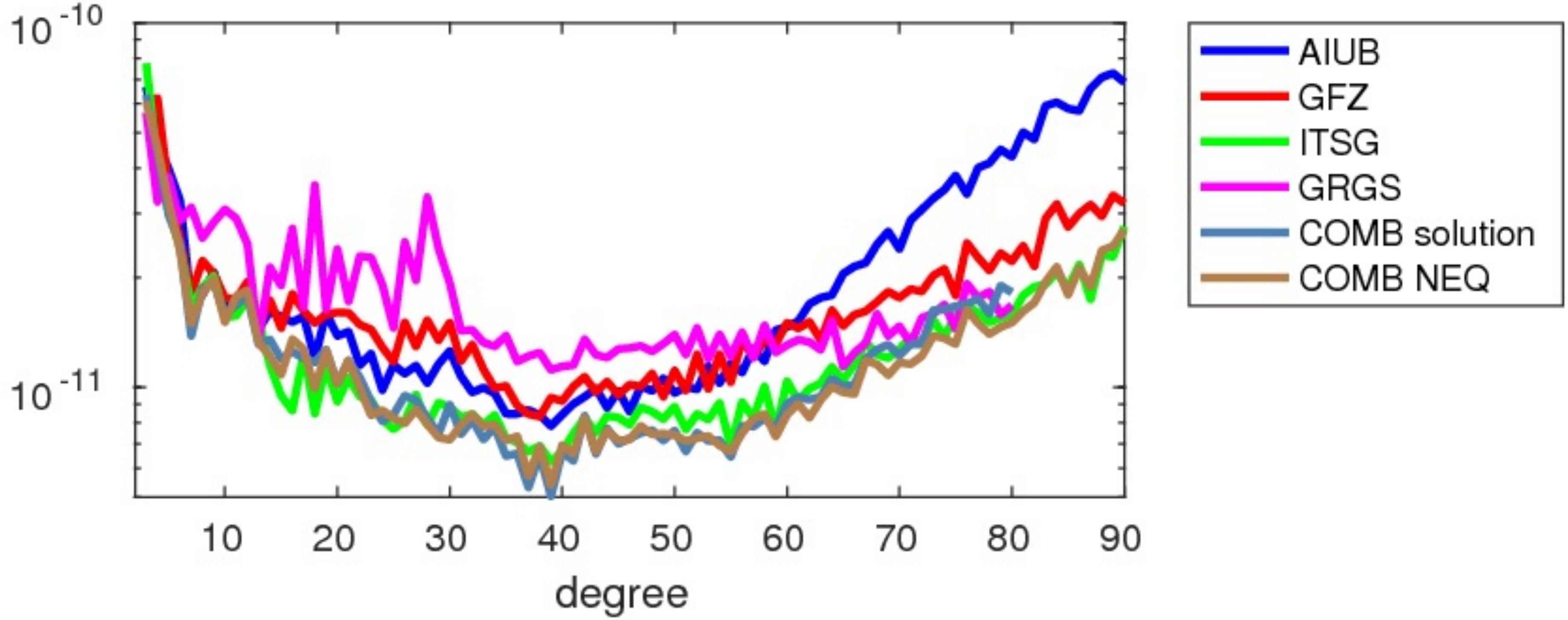
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2006/10



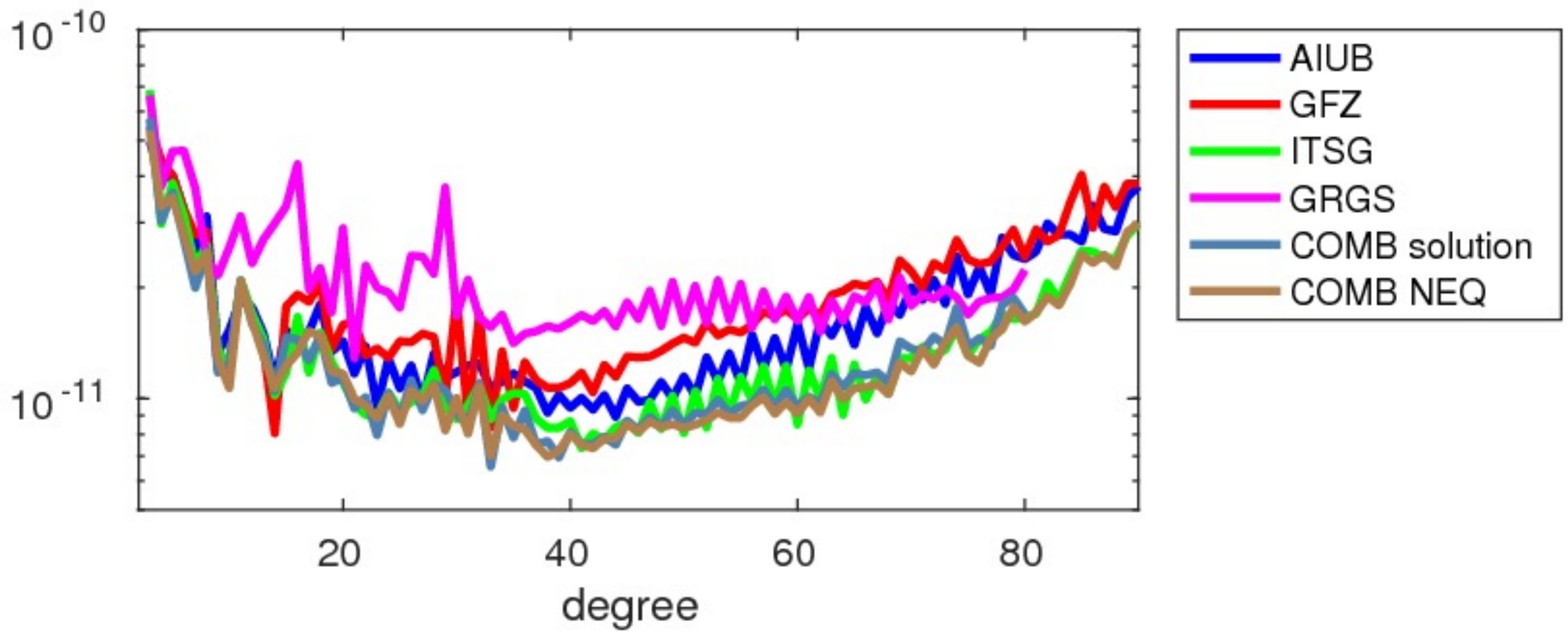
Combination is statistics, not magic -> screening (or improve individual contributions!!)

2006/11



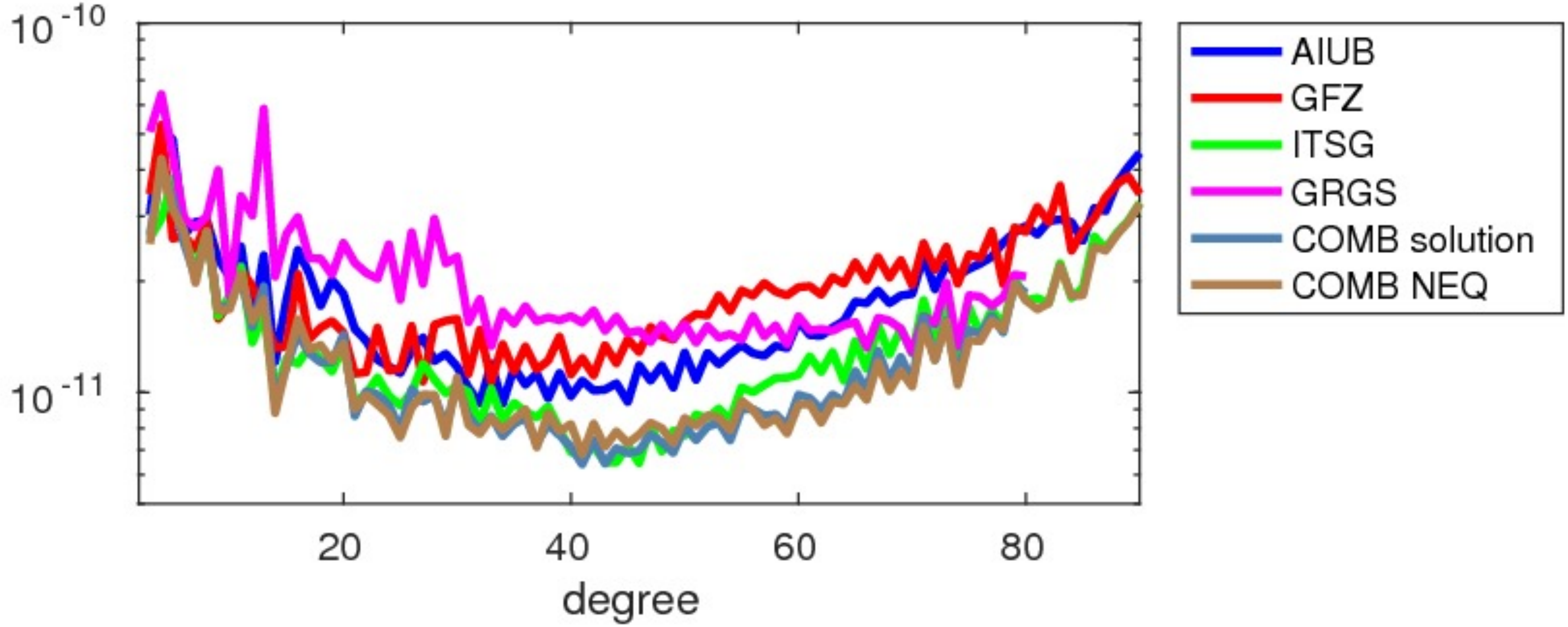
10^{-10}
10^{-11}
10
20
30
40
50
60
70
80
90
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2006/12



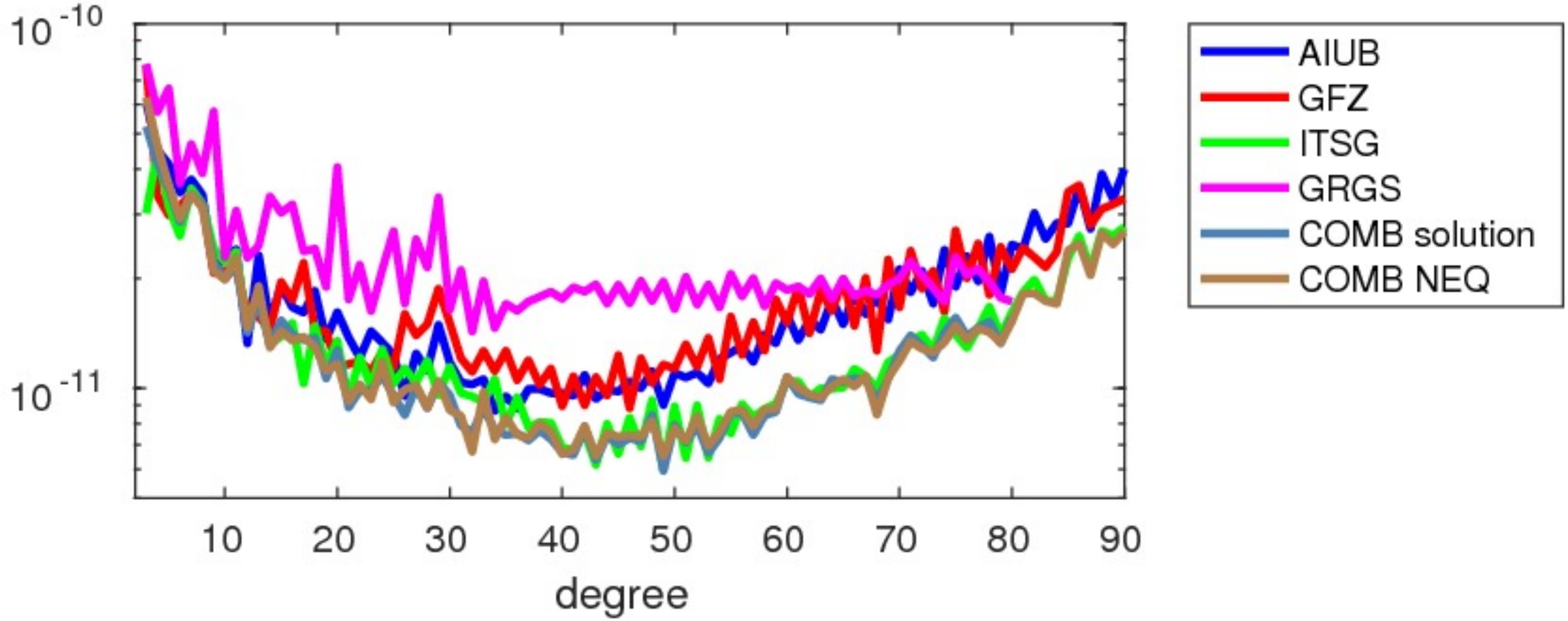
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/01



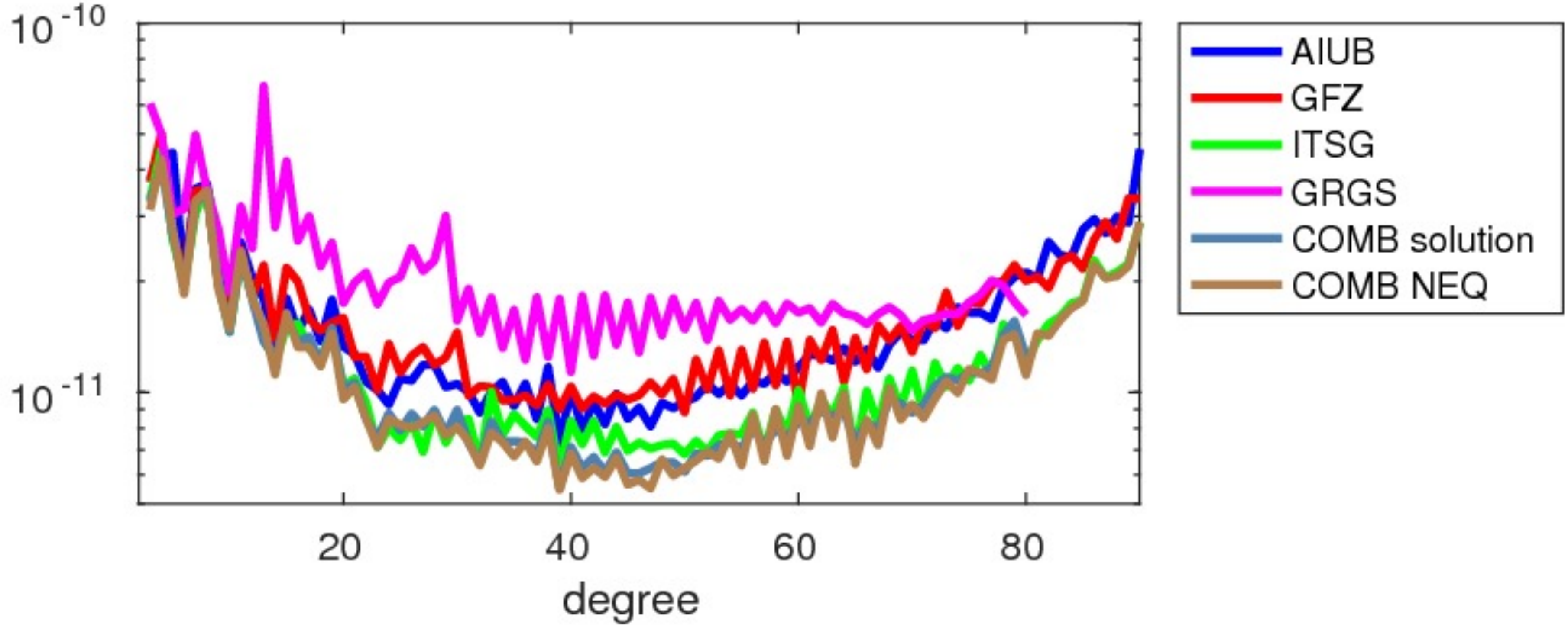
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/02



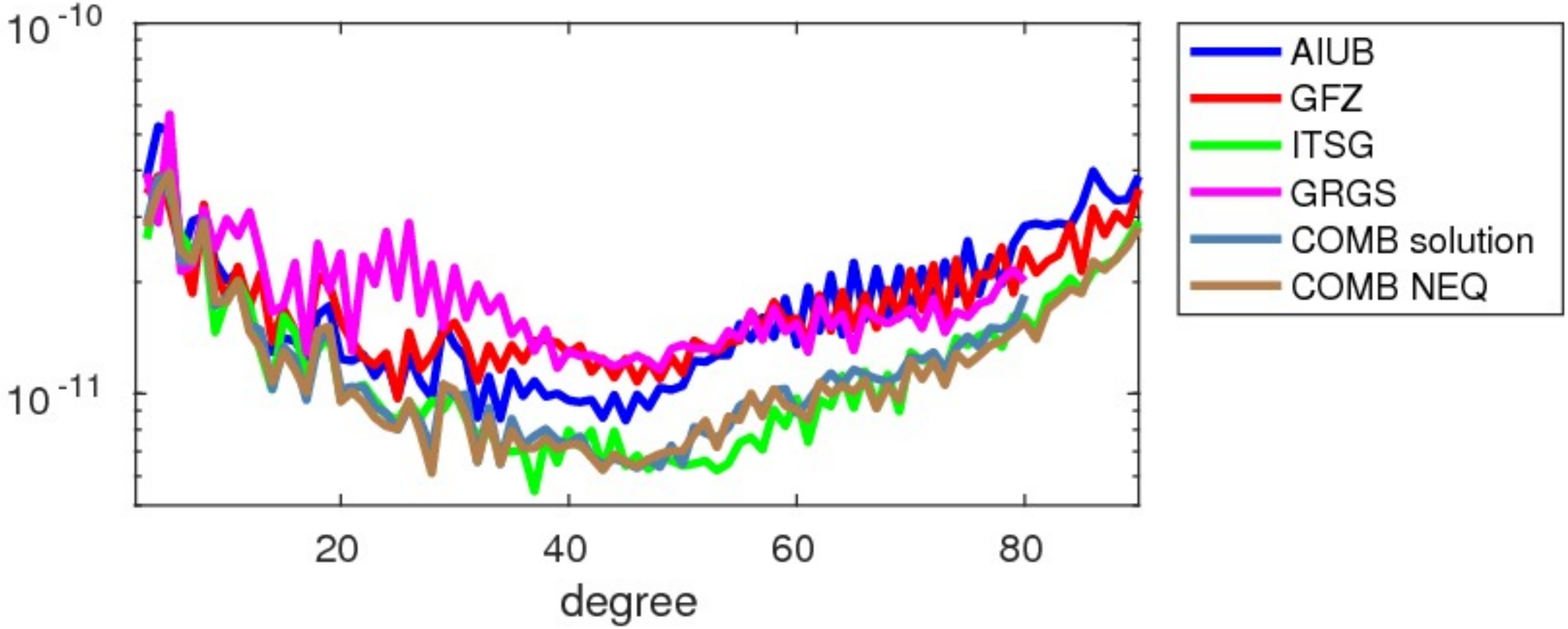
10^{-10}
10^{-11}
10
20
30
40
50
60
70
80
90
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/03



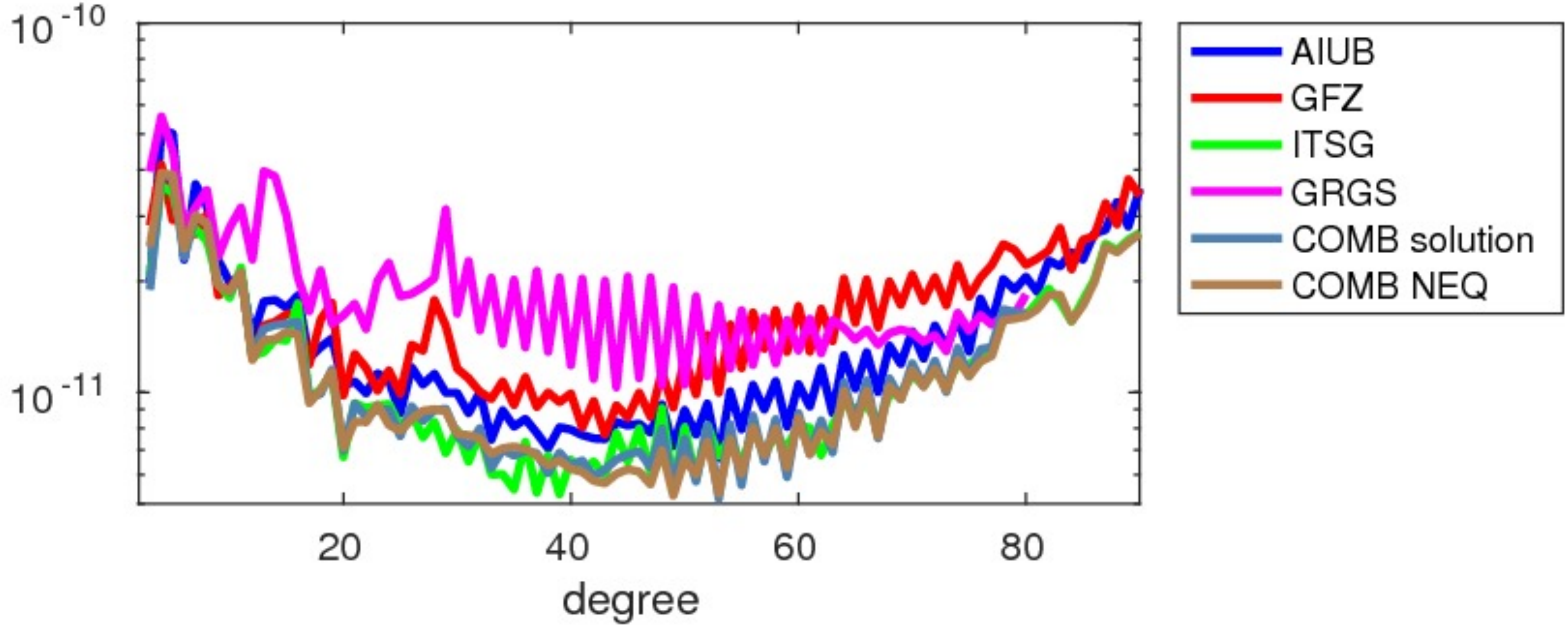
10^-10
10^-11
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/04

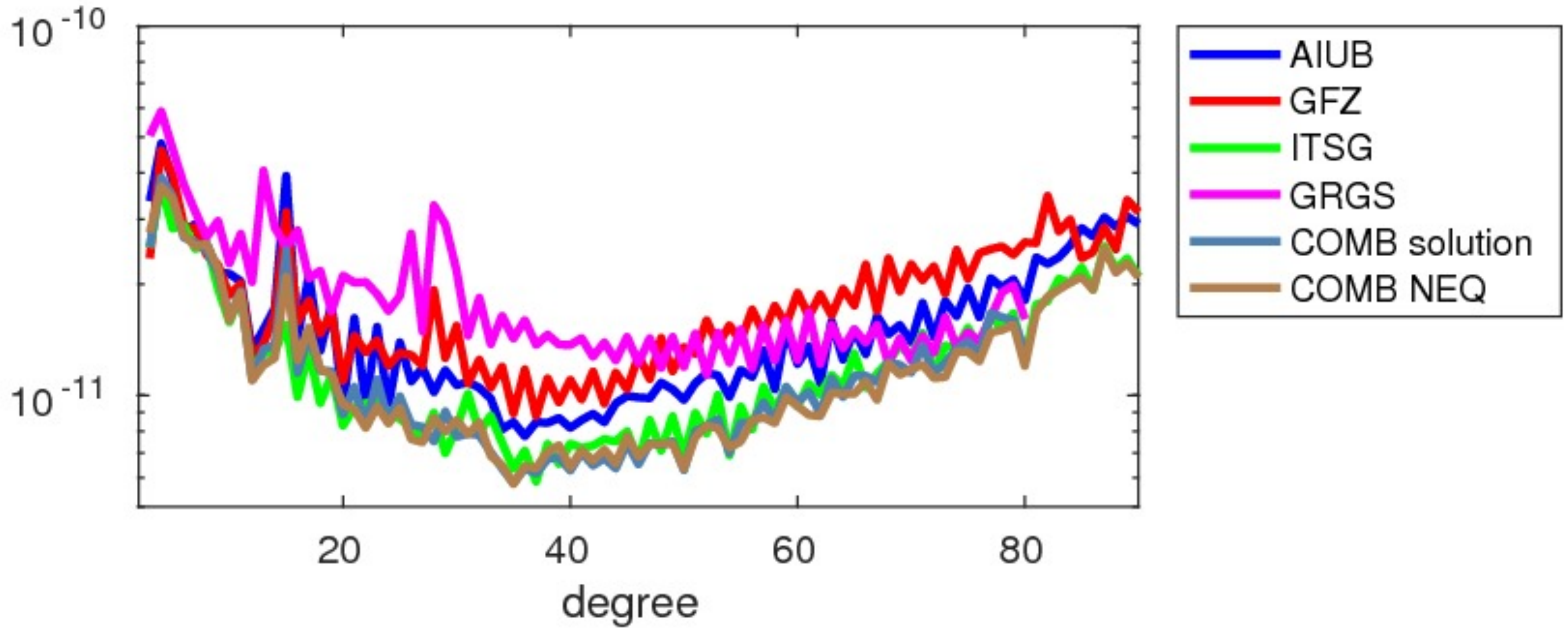


10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/05

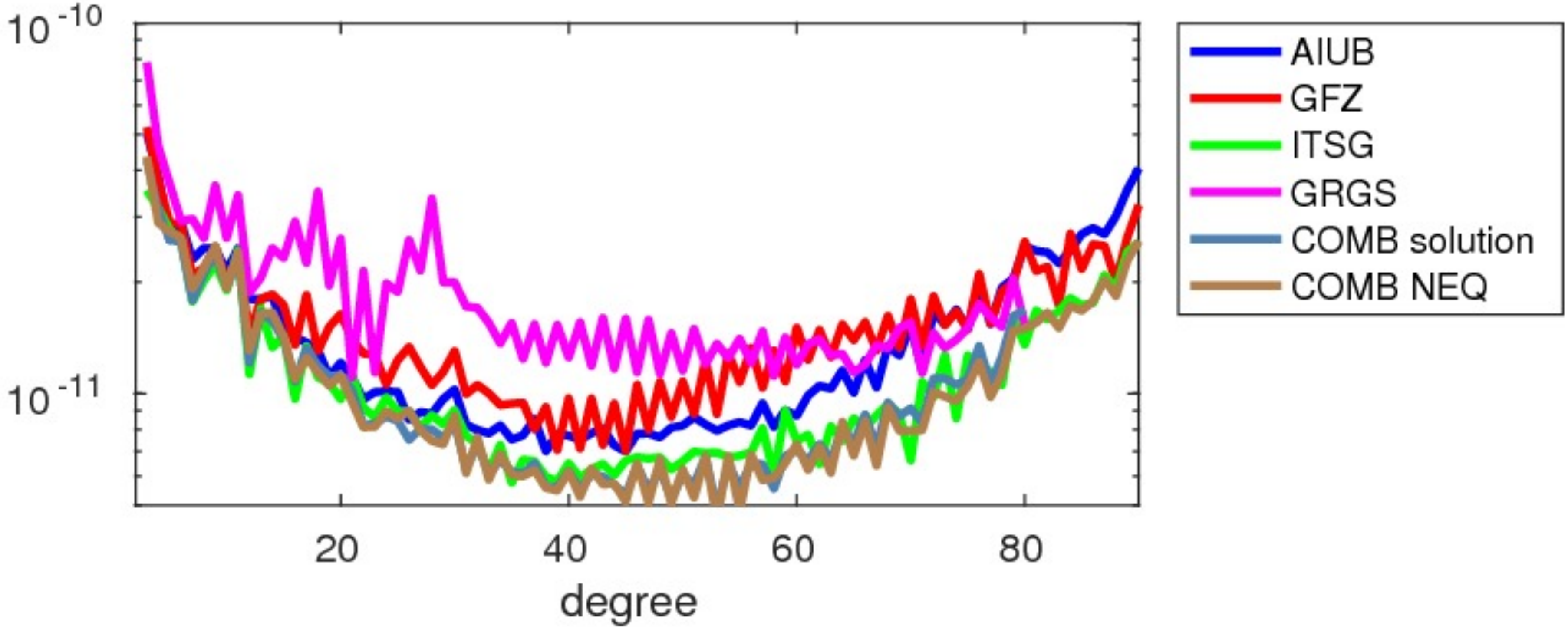

10^-10
10^-11
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/06



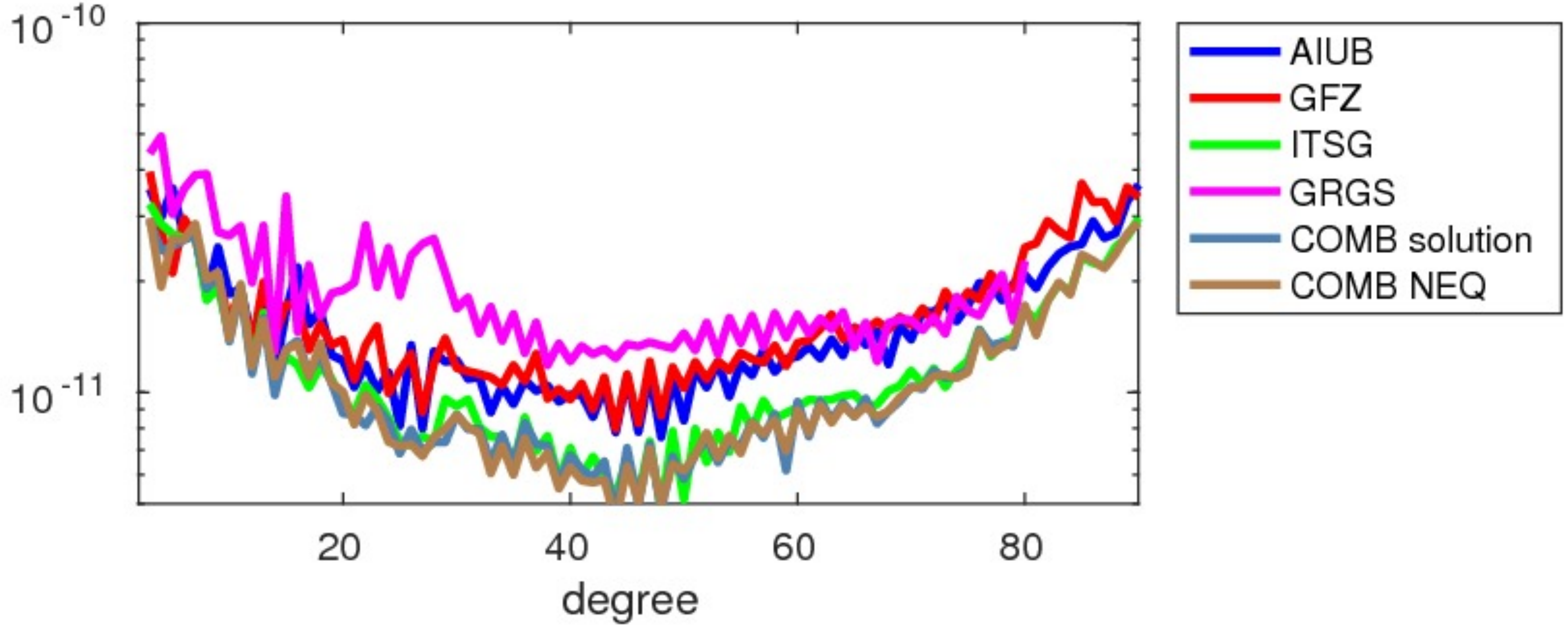
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/07



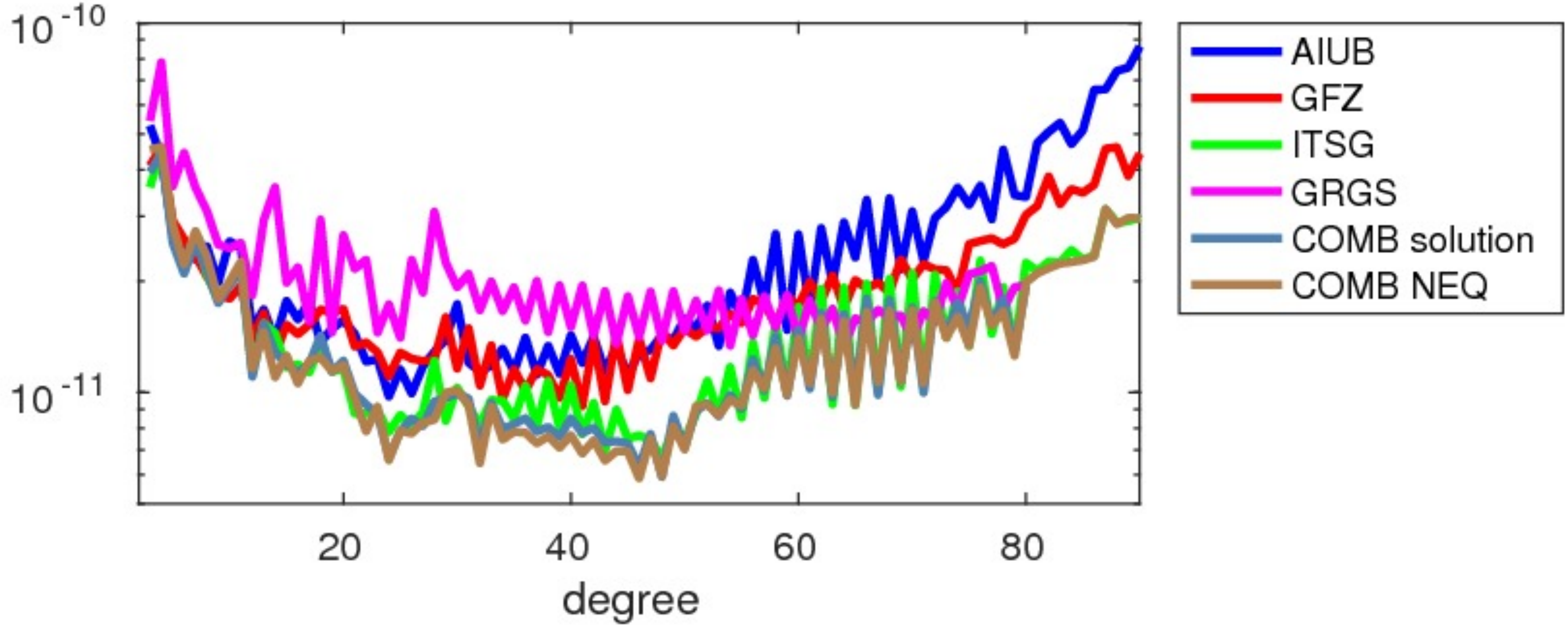
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/08



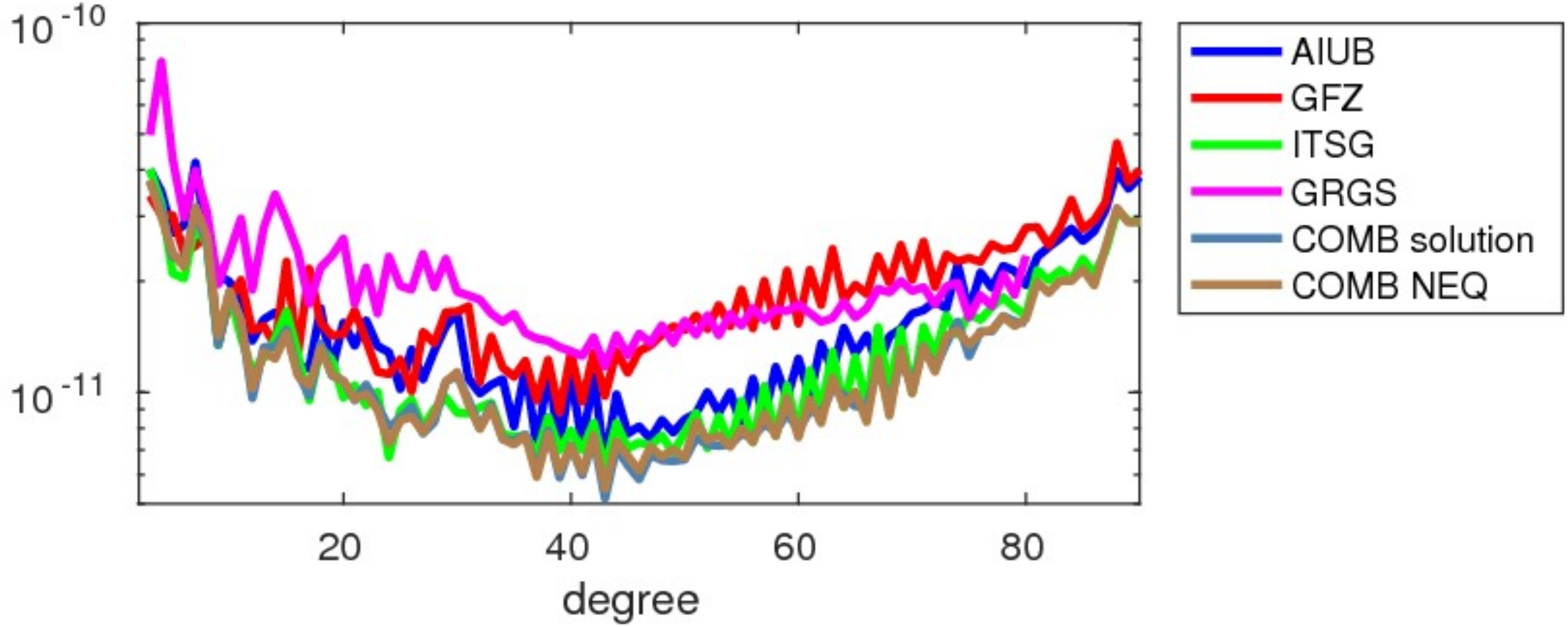
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/09



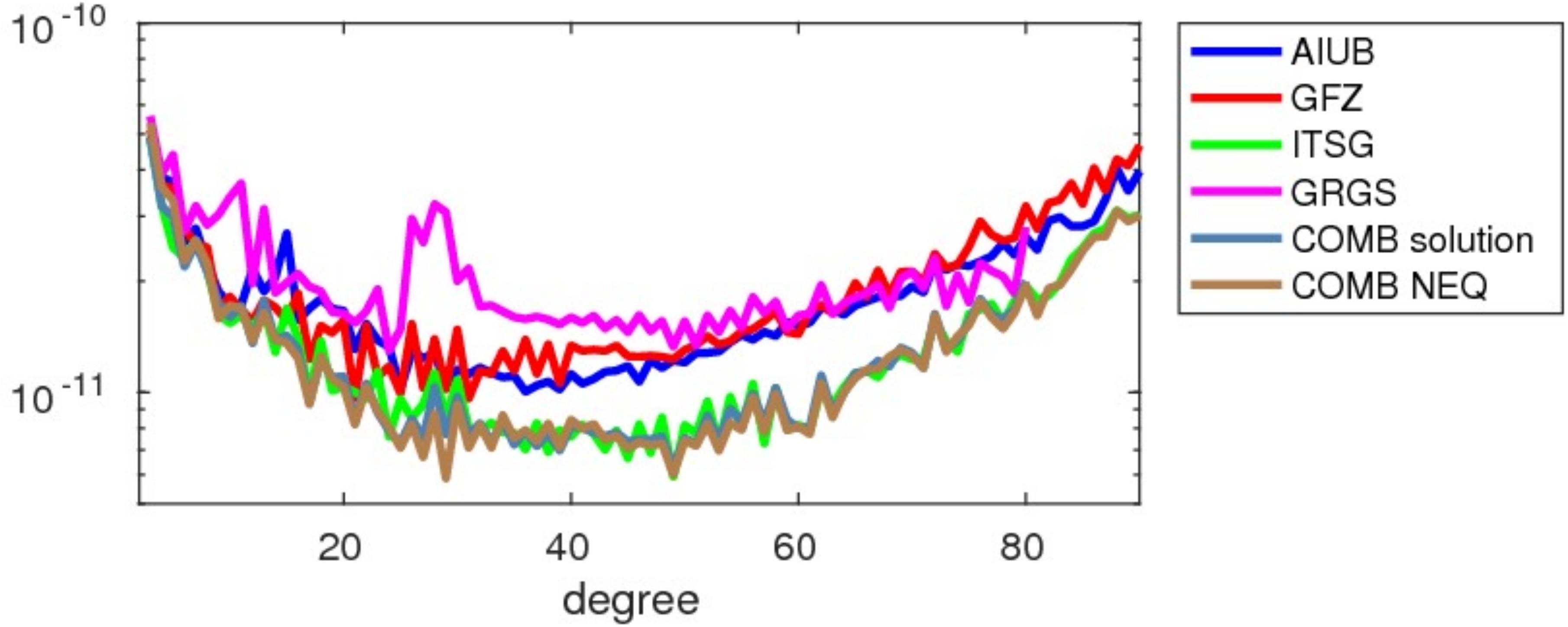
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/10



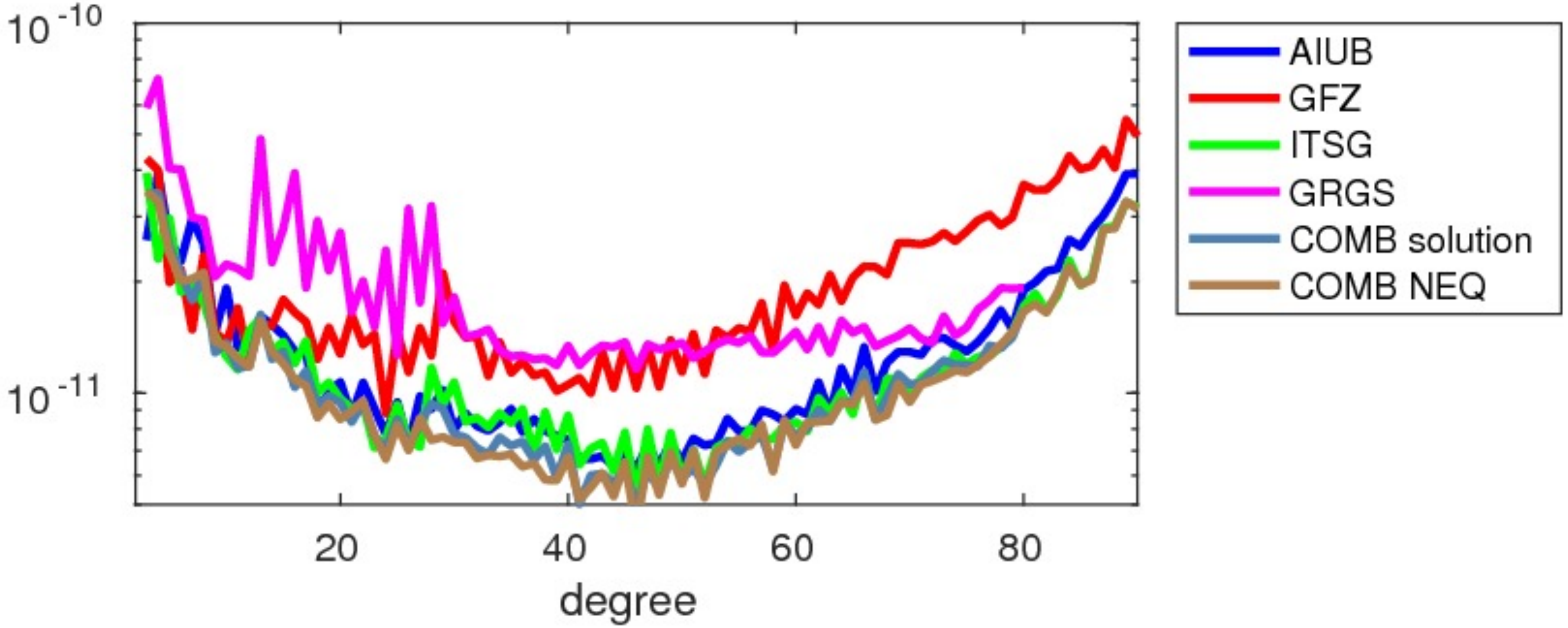
10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/11



10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ

2007/12



10^{-10}
10^{-11}
20
40
60
80
degree
AIUB
GFZ
ITSG
GRGS
COMB solution
COMB NEQ