Estimation of white noise in a mass anomaly time-series and modelling the geocenter motion

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Estimation of white noise

Tikhonov regularization of GRACE-based mass anomaly time-series

Penalty functional:

$$\Phi[H] = \sum_i \left(H(t_i) - H_i^{(\mathrm{obs})} \right)^2 + \alpha \Omega[H]$$

Regularized time-series (to be found)

Original time-series

Regularization parameter

Regularization functional

Regularization functional

Let $H(t) = \begin{pmatrix} h_1(t) \\ h_2(t) \\ \dots \\ h_K(t) \end{pmatrix}$

Mass anomalies in year 1

Mass anomalies in year 2

Mass anomalies in year K

$$\Omega[H] = \sum_{k=1}^{K-1} \int_0^1 \left(\dot{h}_{k+1}(t) - \dot{h}_k(t) \right)^2 dt$$

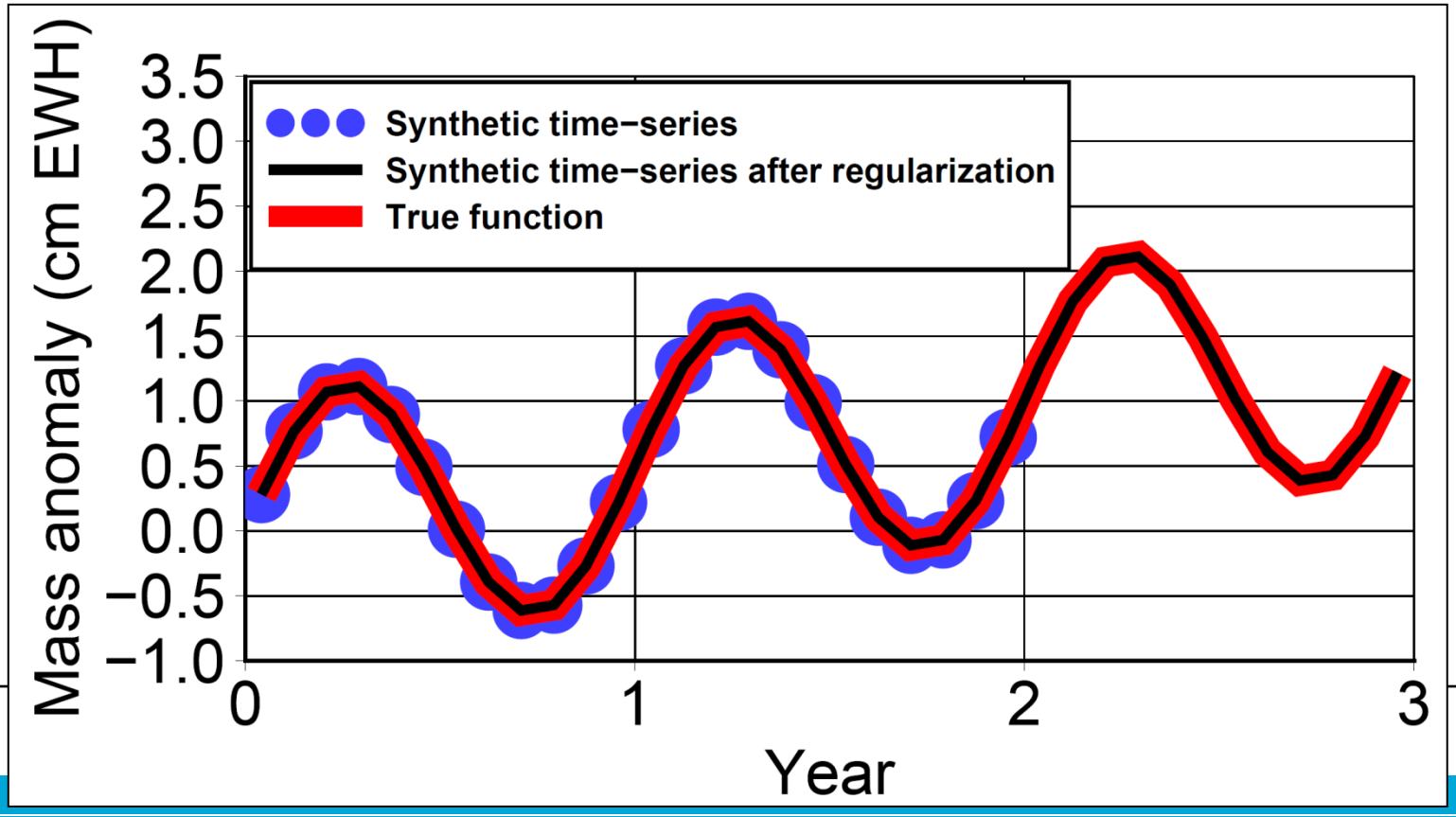
(t – time in years)

Example: Regularization in the absence of noise and penalized signals

$$H(t) = \sin 2\pi t + 0.5 \cdot t$$

t – time in years

$H(t)$ – Equivalent water heights (EWH) in cm



Regularization parameter

$$\alpha = \frac{\sigma^2_{\text{GRACE}}}{\sigma^2_{\text{Signal}}}$$

Noise variance (characterizes random noise in GRACE-based mass anomalies)

Signal variance (characterizes deviations of the actual signal from a non-penalized one)

Variance Component Estimation (see, e.g., Koch & Kusche, JoG, 2002)

Estimation of geocenter motion

Basic idea

- Goal: estimate temporal variations of degree-1 and C_{20} coefficients, as well as the stochastic description of their errors (full covariance matrices)

- Data:
  - GRACE SH coefficients (except for degree-1 and C_{20} coefficients), cleaned from GIA signal
  - Residual OBP estimates (mean monthly values)

- Methodology: statistically-optimal data combination

Statistically-optimal data combination

General format of combining two data sets $\mathbf{d}_1$ and $\mathbf{d}_2$:

$\mathbf{x}_c = (\mathbf{A}_1^T\mathbf{C}_1^{-1}\mathbf{A}_1 + \mathbf{A}_2^T\mathbf{C}_2^{-1}\mathbf{A}_2)(\mathbf{A}_1^T\mathbf{C}_1^{-1}\mathbf{d}_1 + \mathbf{A}_2^T\mathbf{C}_2^{-1}\mathbf{d}_2)$,

$\mathbf{x}_c$ is the re-estimated data set; $\mathbf{A}_{1,2}$ are design matrices; $\mathbf{C}_{1,2}$ are error covariance matrices. In case of combining GRACE data (spectral domain) and OBP data (spatial domain):

$\mathbf{x}_c = (\mathbf{T}^T\mathbf{C}^{-1}\mathbf{T} + \mathbf{S}\mathbf{Y}^T\mathbf{P}\mathbf{Y}\mathbf{S})^{-1}(\mathbf{T}^T\mathbf{C}^{-1}\mathbf{x}_g + \mathbf{S}\mathbf{Y}^T\mathbf{P}\mathbf{h})$.

Note that all the other coefficients are re-estimated.

- ✓ $\mathbf{x}_g$: Vector containing GRACE coefficients.
- ✓ $\mathbf{C}$: Full error covariance matrix of GRACE data.
- ✓ $\mathbf{T}$: Truncated unit matrix.
- ✓ $\mathbf{Y}$: Transformation from spatial to spectral domain.
- ✓ $\mathbf{h}$: Vector containing OBP data.
- ✓ $\mathbf{S}$: Matrix transforming dimensionless coefficients into mass coefficients.
- ✓ $\mathbf{O}$: Ocean function, equals 1 over ocean and 0 over land.
- x $\mathbf{C}_o$: Error covariance matrix of OBP data (diagonal). $(C_o = P^{-1})$

Input data

- GRACE CSR RL05 solutions (including error covariance matrices)
- GIA model of A et al. (2012)
- Noise in AOD1B product (Dobslaw et al, 2015)

Estimation of C_o

(noise is assumed to be stationary)

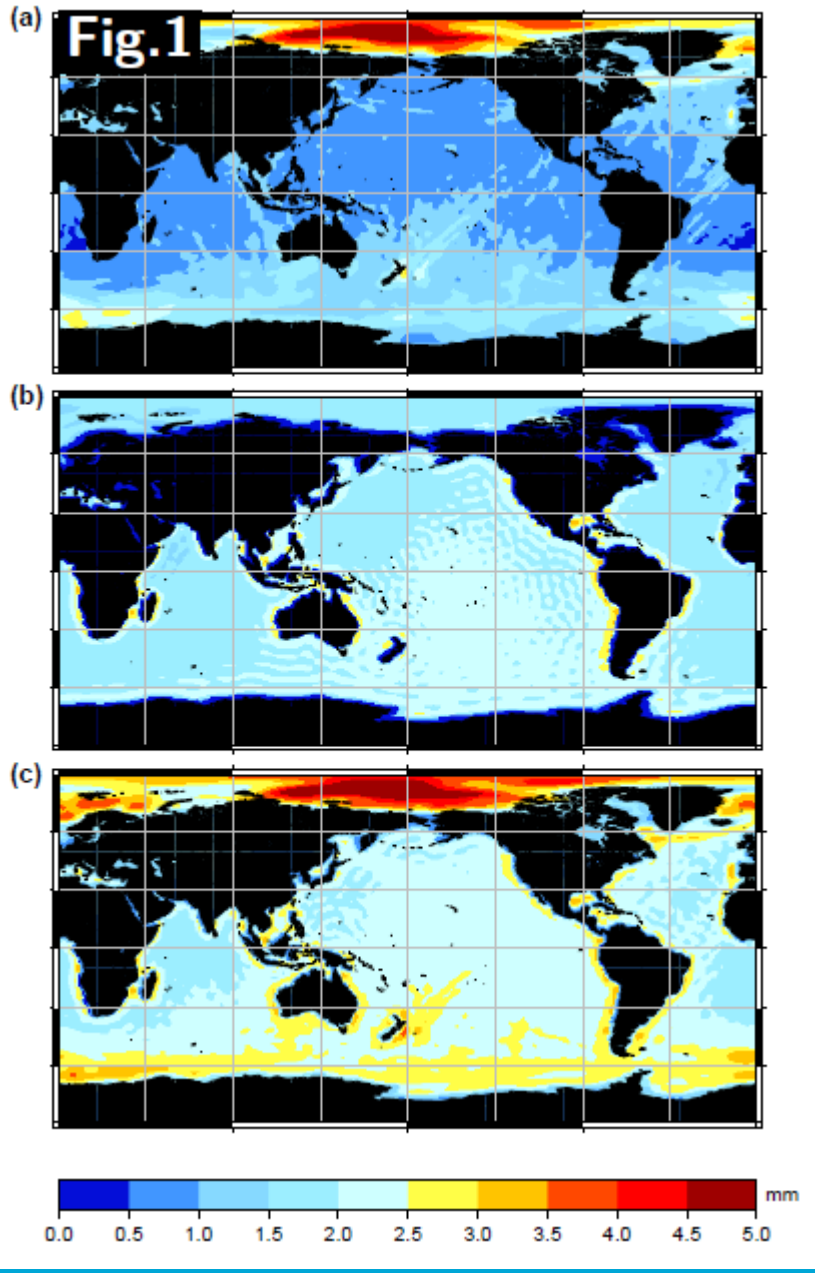
SD of noise in AOD1B product

+

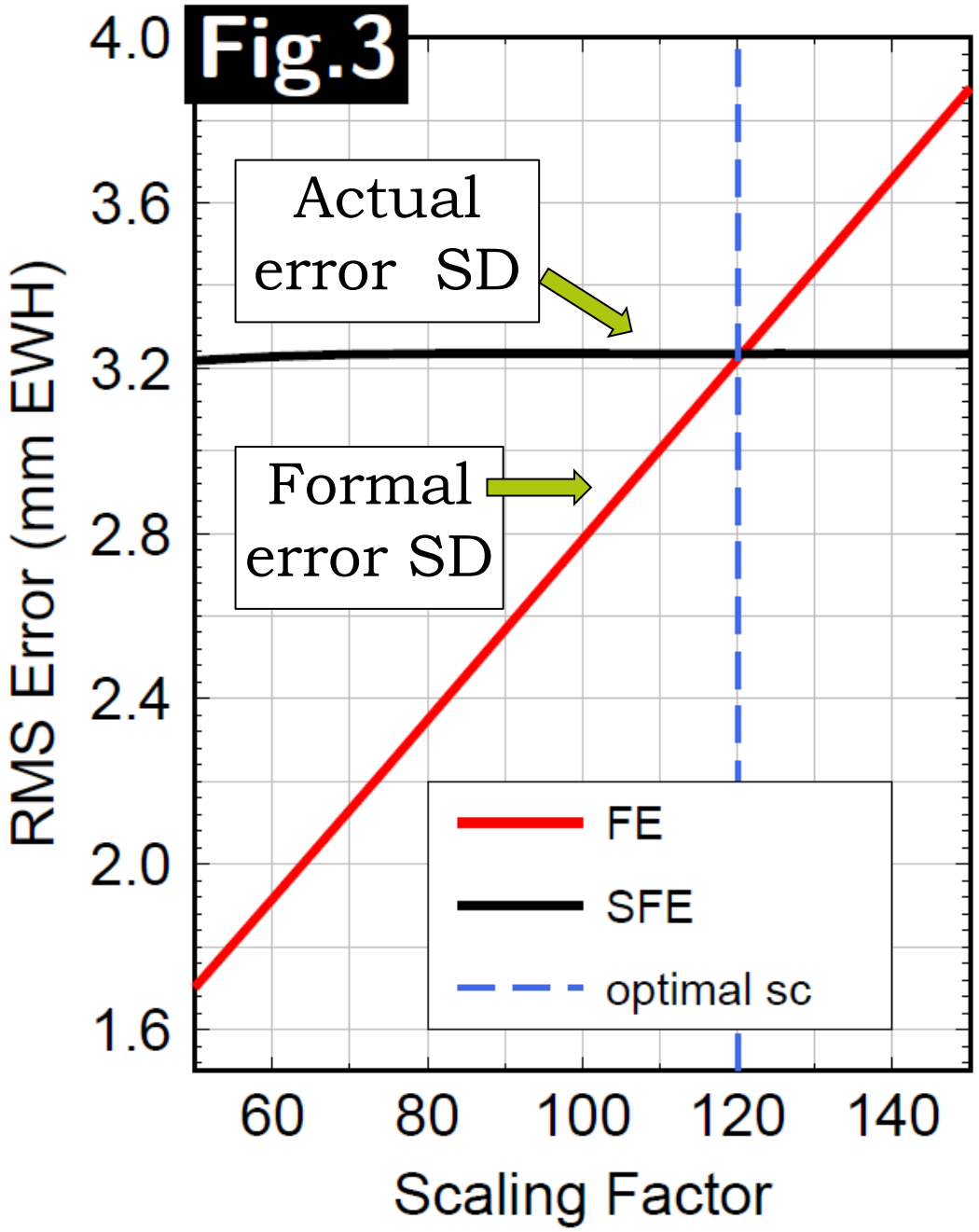
SD of noise in fingerprint estimates (based on GRACE error covariance matrices)

=

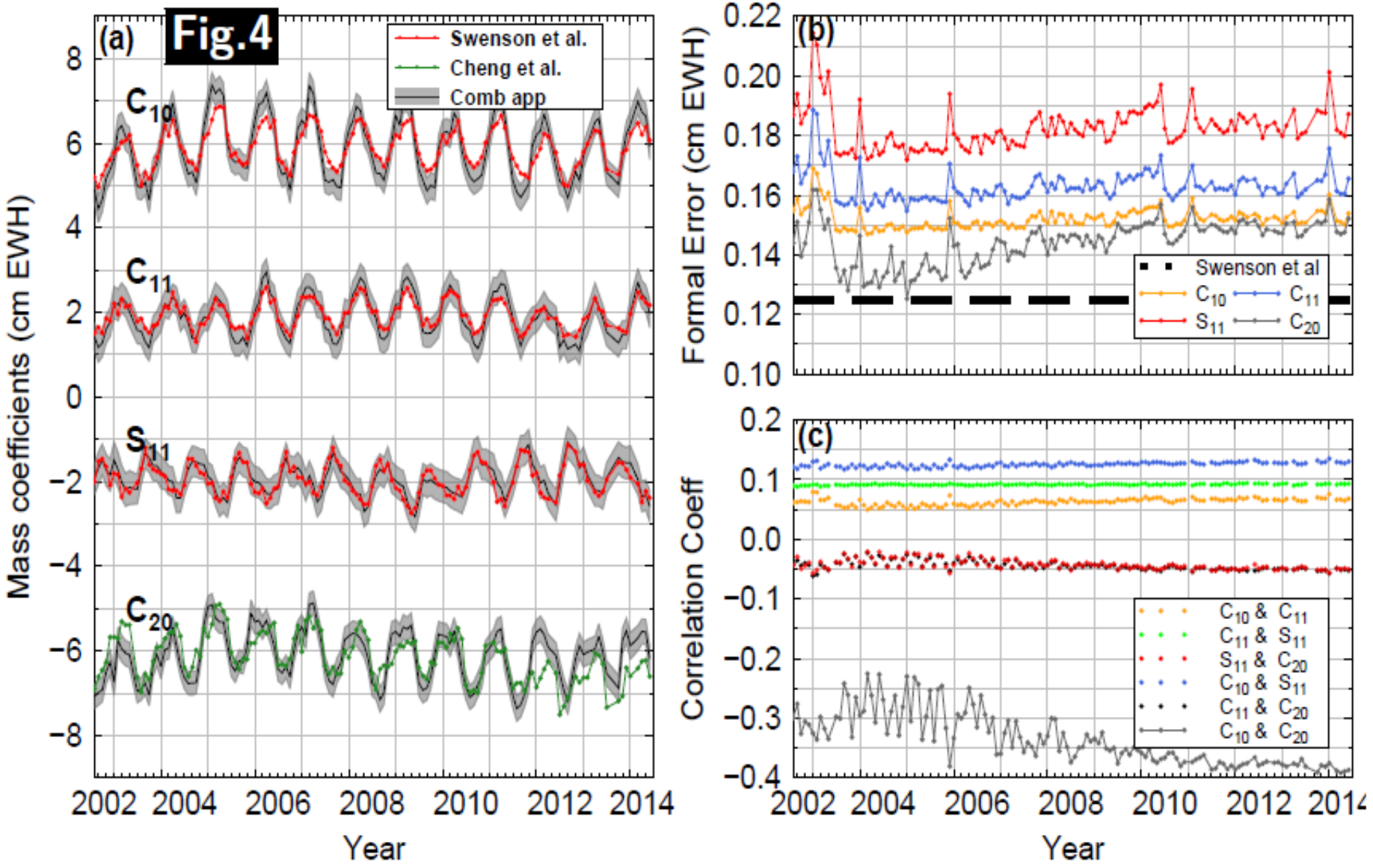
Total noise SD



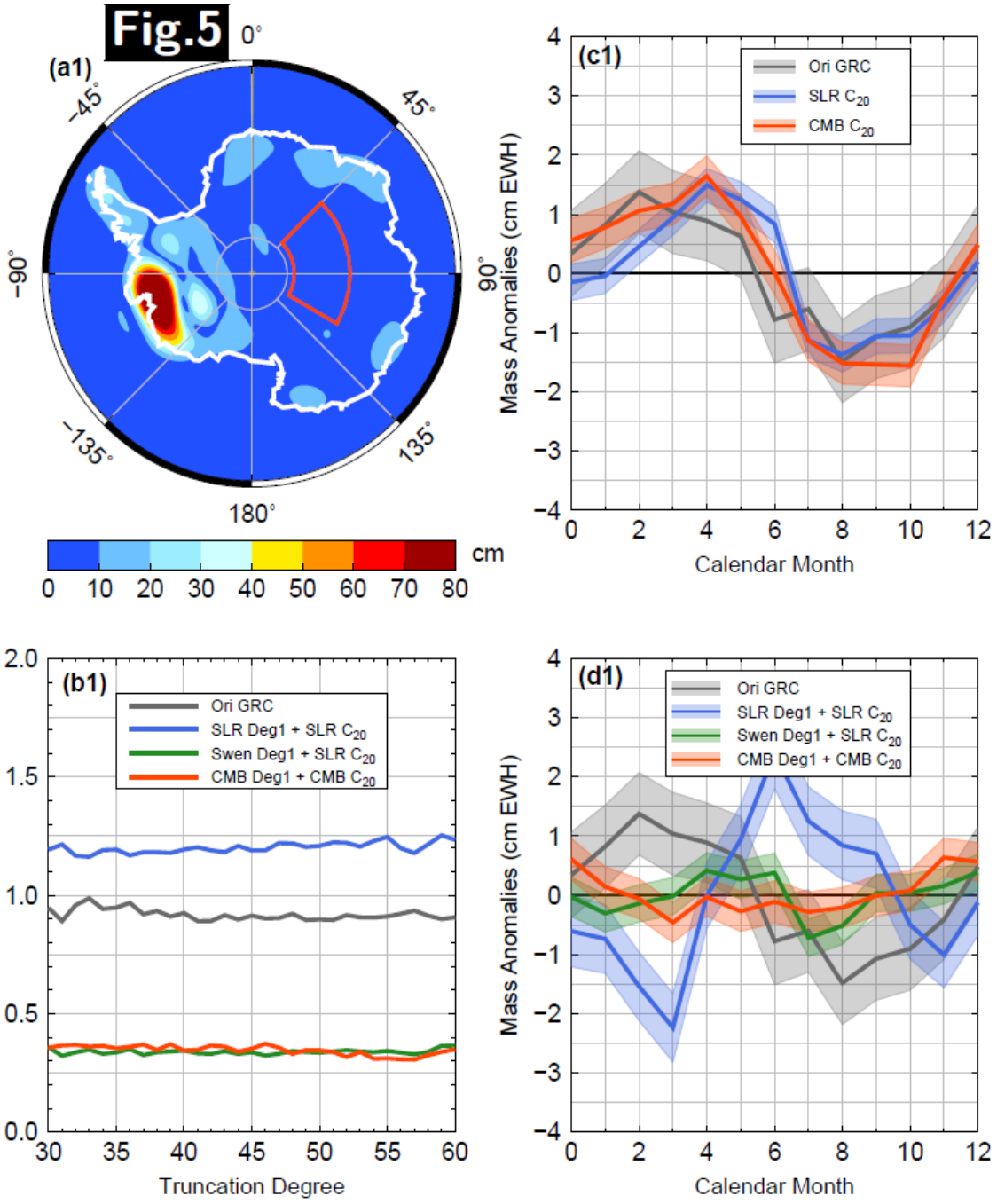
Upscaling of C_o



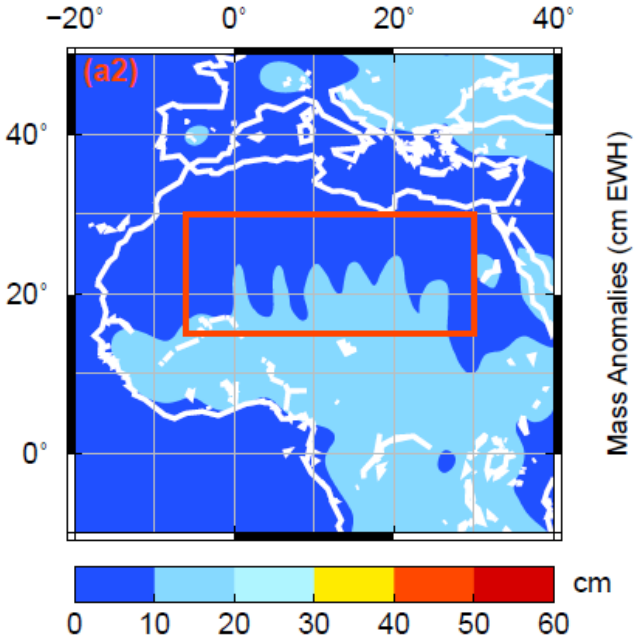
Final product

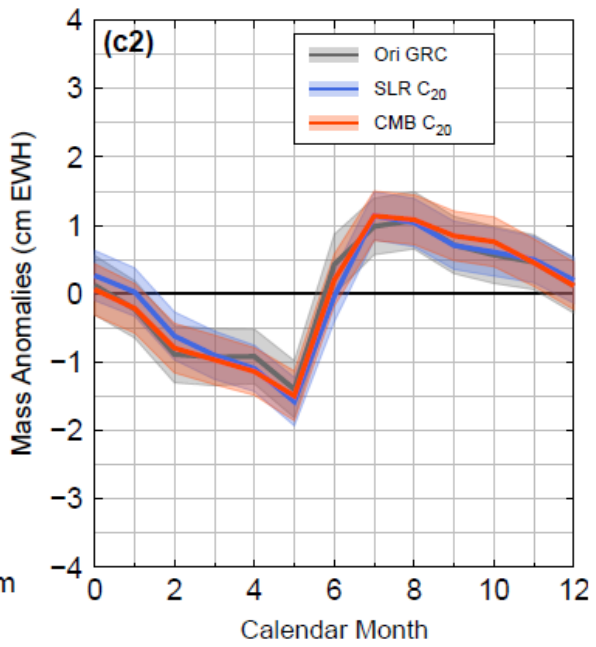


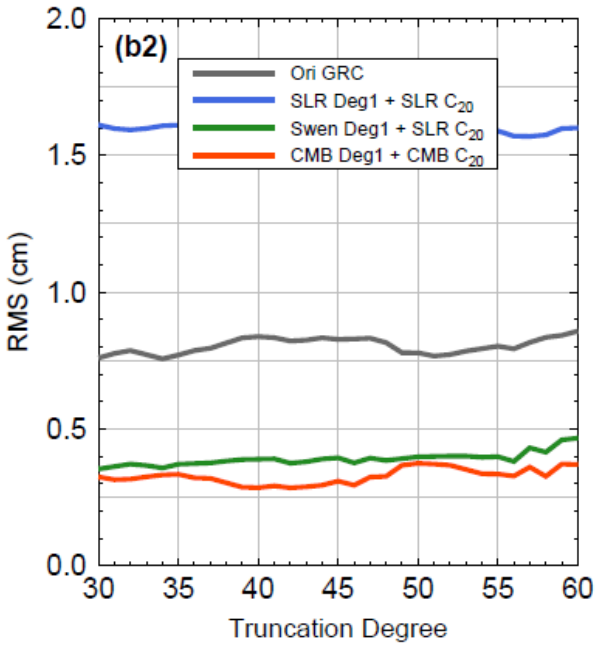
Validation: East Antarctica

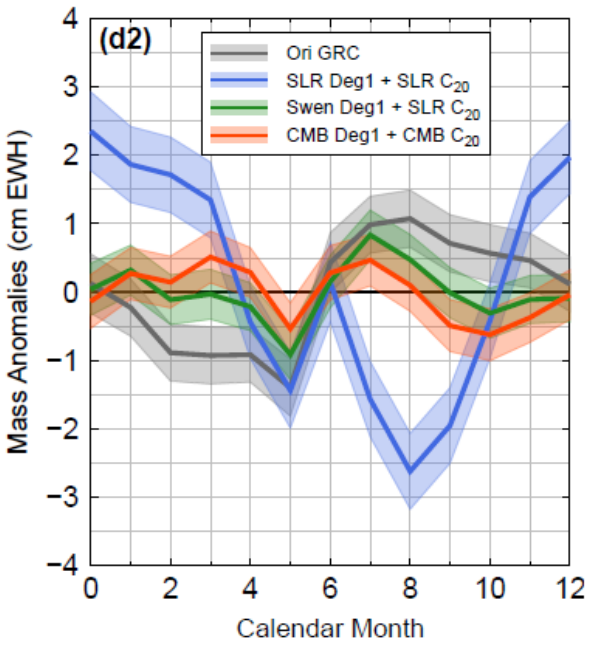


Validation: Sahara









The produced time-series of degree-1 and C_{20} coefficients will be available from:

http://www.citg.tudelft.nl/deg1&c20