EGSIEM

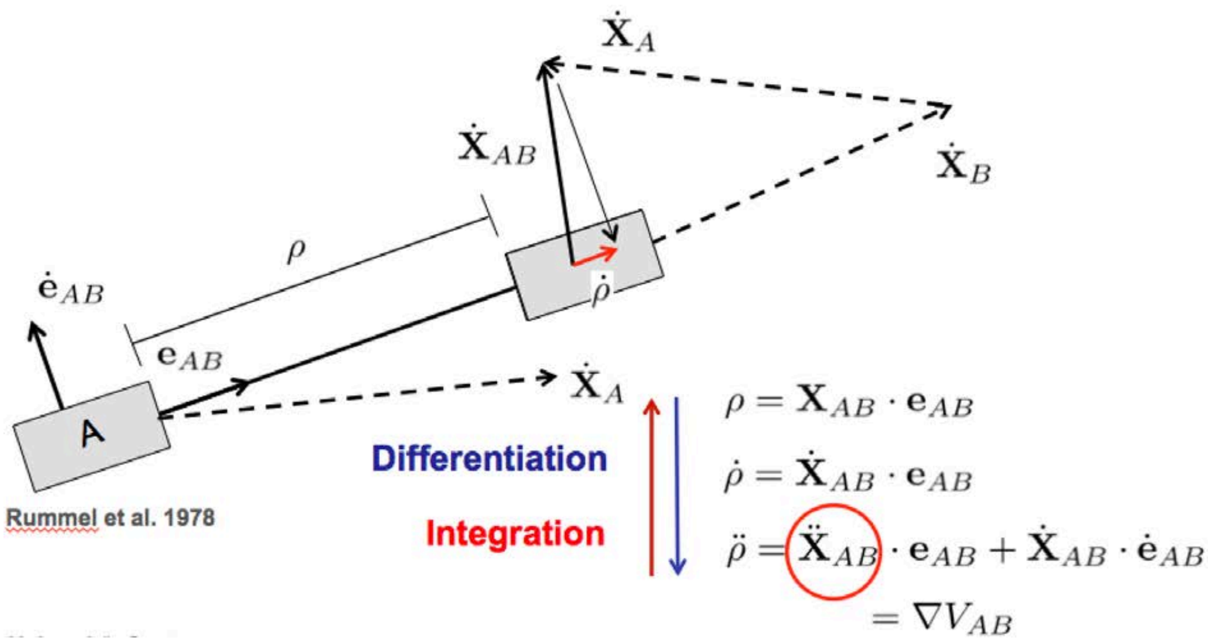
European Gravity Service for Improved Emergency Management

Implementation of the rigorous acceleration approach

Ulux progress on WP2

The acceleration approach-an alternative way of processing GRACE data

– Concept: link kinematic observations to forces based on Newton‘s equation of motion



$$\nabla V_{AB} \cdot \mathbf{e}^{a}_{AB} = \ddot{\rho} - \frac{1}{\rho}\left(\dot{\mathbf{x}}_{AB} \cdot \dot{\mathbf{x}}_{AB} - \dot{\rho}^2\right)$$

$10\ \mathrm{nm}/\mathrm{s}^2$ $1\ \mu\mathrm{m}$ $0.1\ \mathrm{mm}/\mathrm{s}$ $0.1\ \mu\mathrm{m}/\mathrm{s}$

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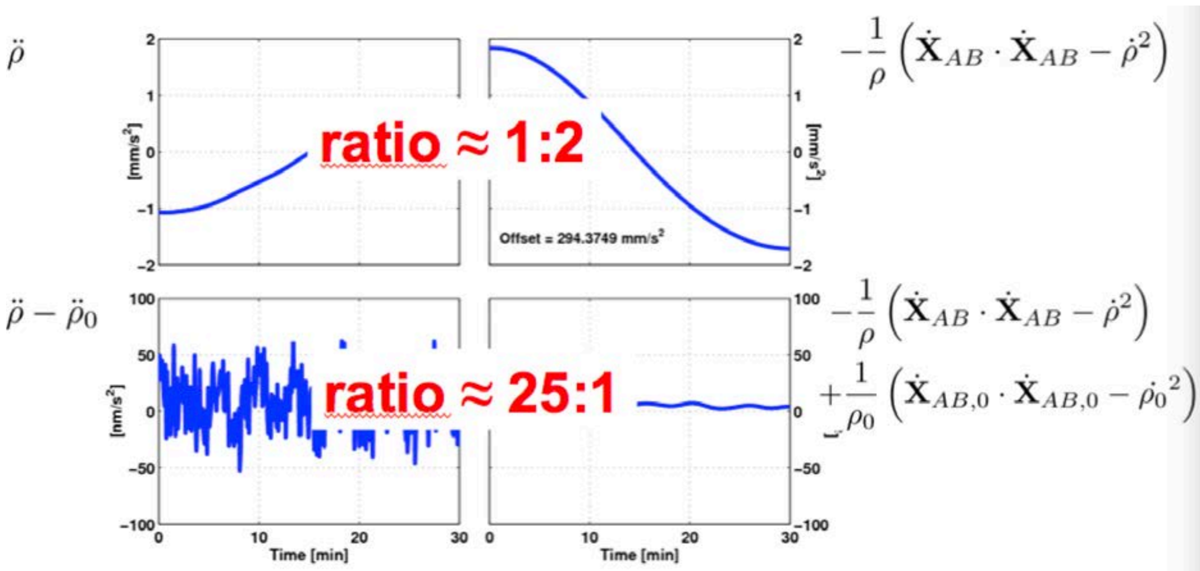
The acceleration approach-an alternative way of processing GRACE data

– Advantage
  - No accumulation of numerical integration errors or model errors
  - Allows for a point-wise application (especially suitable for regional and local applications)
– Downside
  - requires range acceleration and centrifugal component (GPS observations)

Approximate solution

– Reducing the observations to residual quantities



– Neglecting the residual terms

$$\ddot\rho - \ddot\rho^0 \approx (\nabla V_B - \nabla V_A)\cdot \mathbf{e}^{\mathrm{a}}_{\mathrm{AB}} - (\nabla V_B^0 - \nabla V_A^0)\cdot \mathbf{e}^{\mathrm{a},0}_{\mathrm{AB}}$$

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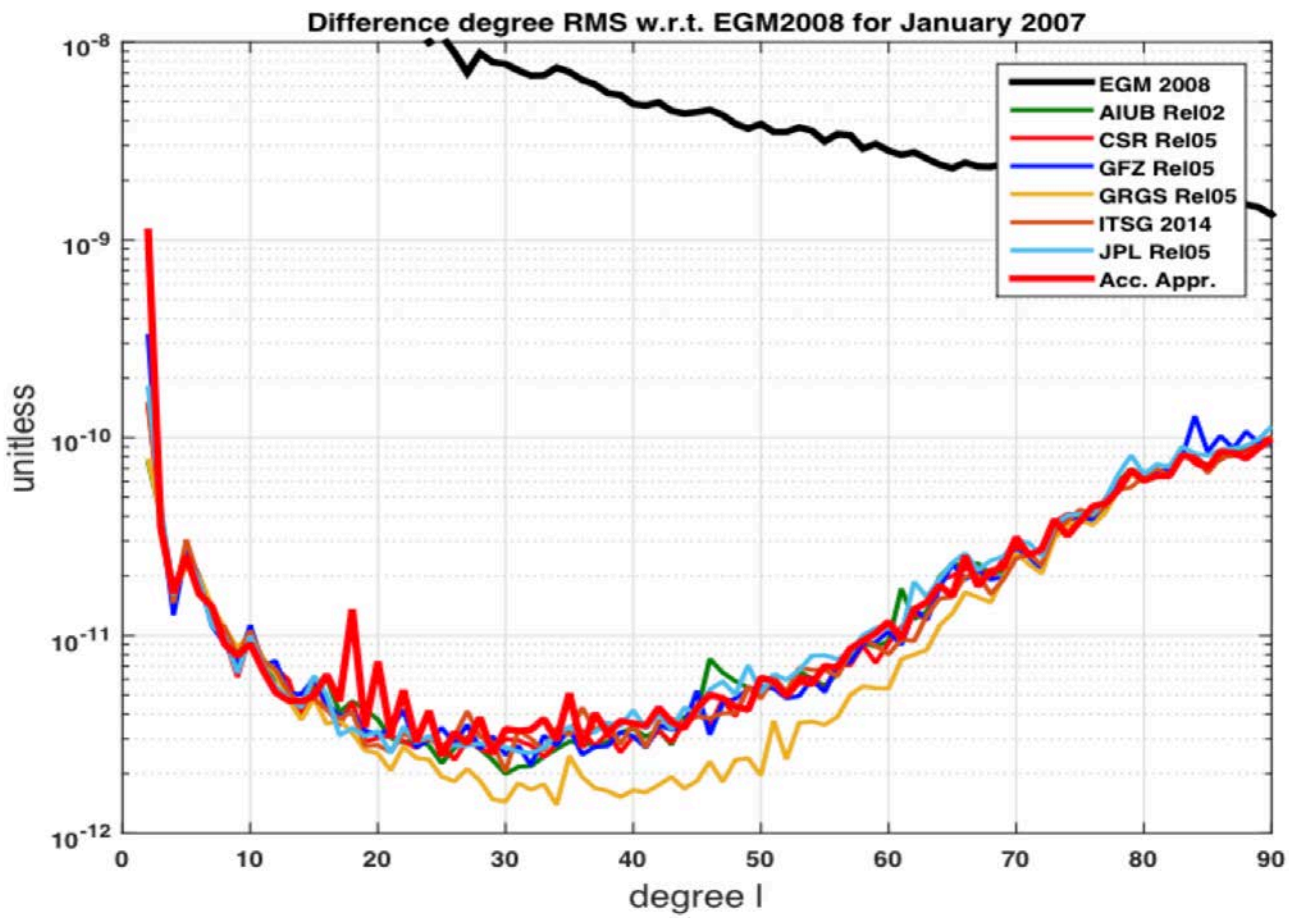
Approximate solution



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Rigorous solution

- Assumption degrades the solution
- Error at degree 2 and around degree 16 (number of revolutions per day for GRACE)
- Improvement: Rigorous approach

$$
\begin{aligned}
\ddot{\rho} &= \ddot{\mathbf{x}}_{\mathrm{AB}} \cdot \mathbf{e}^{\mathrm{a}}_{\mathrm{AB}} &+ \frac{1}{\rho}\dot{\mathbf{x}}_{\mathrm{AB}} \cdot \dot{\mathbf{x}}_{\mathrm{AB}} &- \frac{\dot{\rho}^2}{\rho} \\
&= (\nabla V_B - \nabla V_A) \cdot \mathbf{e}^{\mathrm{a}}_{\mathrm{AB}} &+ \frac{1}{\rho}\dot{\mathbf{x}}_{\mathrm{AB}} \cdot \dot{\mathbf{x}}_{\mathrm{AB}} &- \frac{\dot{\rho}^2}{\rho} \\
&= f &+ g_1 &+ g_2
\end{aligned}
$$

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Rigorous solution

- Introduce a priori observation and reduce the equation system to residual quantities (fit an “Observed“ orbit and approximate a dynamic orbit)

$$\ddot{\rho} - \ddot{\rho}^0 = (\nabla V_B - \nabla V_A) \cdot \mathbf{e}_{\mathrm{AB}}^{\mathrm{a}} + \frac{1}{\rho}\left(\dot{\mathbf{x}}_{\mathrm{AB}} \cdot \dot{\mathbf{x}}_{\mathrm{AB}} - \dot{\rho}^2\right)$$
$$- \left(\nabla V_B^0 - \nabla V_A^0\right) \cdot \mathbf{e}_{\mathrm{AB}}^{\mathrm{a},0} - \frac{1}{\rho^0}\left(\dot{\mathbf{x}}_{\mathrm{AB}}^0 \cdot \dot{\mathbf{x}}_{\mathrm{AB}}^0 - \left(\dot{\rho}^0\right)^2\right)$$

- Linearize the right-hand side of the above equation

$$\ddot{\rho} - \ddot{\rho}^0 \approx \sum_i \frac{\partial f}{\partial p_i}\Delta p_i + \sum_i \frac{\partial g_1}{\partial p_i}\Delta p_i + \sum_i \frac{\partial g_2}{\partial p_i}\Delta p_i$$

$$(\nabla V_B - \nabla V_A) \cdot \mathbf{e}_{\mathrm{AB}}^{\mathrm{a}} - \left(\nabla V_B^0 - \nabla V_A^0\right) \cdot \mathbf{e}_{\mathrm{AB}}^{\mathrm{a},0} = \sum_i \frac{\partial f}{\partial p_i}\Delta p_i + \hbar^2$$
$$\frac{1}{\rho}\dot{\mathbf{x}}_{\mathrm{AB}} \cdot \dot{\mathbf{x}}_{\mathrm{AB}} - \frac{1}{\rho^0}\dot{\mathbf{x}}_{\mathrm{AB}}^0 \cdot \dot{\mathbf{x}}_{\mathrm{AB}}^0 = \sum_i \frac{\partial g_1}{\partial p_i}\Delta p_i + \hbar^2$$
$$-\frac{\dot{\rho}^2}{\rho} + \frac{\left(\dot{\rho}^0\right)^2}{\rho^0} = \sum_i \frac{\partial g_2}{\partial p_i}\Delta p_i + \hbar^2,$$

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Rigorous solution

– Solve the variational equations using the variation of constant approach (Jäggi, 2007)
  - Variational equation for the initial conditions (homogeneous solution)

$$\frac{d}{dt}\begin{pmatrix}\Phi \\ \dot{\Phi}\end{pmatrix} = \begin{pmatrix}\dot{\Phi} \\ F(\mathbf{x},\dot{\mathbf{x}})\cdot\Phi\end{pmatrix}$$

  - Variation of constants (inhomogeous solution)

$$\boldsymbol{\alpha}_{p_i}(t) = \int_{t_0}^{t} \Phi^{-1}(\tau)\cdot\frac{\partial\mathbf{h}(\tau)}{\partial p_i}d\tau$$

$$\boldsymbol{\phi}_{p_i}(t) = \Phi(t)\cdot\boldsymbol{\alpha}_{p_i}(t)\,.$$

– Connect the above derivatives to the linearized mathematical model by applying the chain rule

Rigorous solution

- Belongs to another implimentation of the variational equations
- Theoretically give identical results as the other approaches implemented within the EGSIEM project
- Practically different processing schemes come with their particular advantages and disadvantages. By combining the different solutions would be able to benefit from the advantages and to mitigate the disadvantages.

Current implementation status at UL

- **Already finished:**
  - **Data screening for Grace A and B using GNV1B data**
  - **Testing the covariance information of Grace A and B for positive definiteness**
  - **Correcting the non-linear behavior in the accelerometer data**
  - **synchronization of the GRACE A and B**
  - **Orbit adjustment for Grace A and B (done, but may have problems…)**
- **Under going:**
  - **Combined adjustment of Grace A, B and K-Band data (Stucked)**

Problems to be solved

- **Orbit adjustment:**
  - **Different iterations on different computers (result in different input orbit for later combined adjustment):**
    - **Desktop (matlab R2015b), stopped at the 3rd iteration**
    - **Mac (matlab R2014b), stopped at the 5th iteration**
    - **Different initial conditions reaching 1cm**
- **Combined adjustment:**
  - **Different N matrix for range bias (3), acc scale (3) and bias (3), I think both are wrong, since there are negative diagal values**
    - **Desktop, acc scale negative**
    - **Mac, acc bias negative**
  - **Different b (observed range-xAB)**

Problems to be solved

13.7910	−13.8353	0.1207	1.7464	1.3008e...	−0.0036	2.5945e...	0.0332	−1.3877e...
−13.8368	31.0844	−17.2634	0.1868	−1.2647e...	5.1209e−...	7.2726e...	−0.0125	1.7153e...
0.1535	−17.2606	17.0356	−0.0962	−1.9881e...	0.0042	8.1848e...	6.2128e−...	7.7294e...
1.5583	0.1901	−0.0559	9.8670	2.9159e...	−0.0885	−1.2257e...	0.1573	−1.4768e...
7.7435e...	−1.2291e...	−3.3769e...	2.9861e...	−1.0137e...	2.0991e...	9.5364e...	2.6439e...	5.2431e...
−0.0016	3.7908e−...	0.0069	−0.0811	2.0238e...	3.0603e...	−2.0963e...	−7.0171e...	−9.1545e...
−7.0405e...	−1.5969e...	−5.7634e...	2.0623e...	−3.4731e...	8.0361e...	3.0603e...	2.1586e...	1.6700e...
0.0483	−0.0117	0.0066	0.2000	7.9280e...	−0.0023	−9.2404e...	3.0603e...	−3.9403e...
2.3891e...	2.9537e...	2.4107e...	1.0134e...	4.7995e...	−1.2651e...	−6.7151e...	1.4826e...	3.0603e...

					6	7	8	9
14.2034	-13.8100	0.3611	1.6289	1.6609e+05	-0.0398	-4.7538e+05	0.0441	-7.5025e+07
-13.8004	31.0866	-17.2423	0.1813	795.2148	-0.0027	-5.7141e+04	-0.0114	-4.7882e+06
0.4269	-17.2438	17.1961	-0.1428	8.1739e+04	-0.0201	-4.0645e+05	0.0082	-4.1503e+07
-0.7945	0.1786	-0.1666	-3.4022	-1.4276e+05	0.0398	1.6883e+05	-0.0521	7.0000e+07
1.3187e+05	-933.4834	7.4365e+04	-1.2214e+05	4.8220e+10	-1.1095e+04	-2.4011e+11	2.1811e+03	-2.2768e+13
-0.0301	-0.0020	-0.0160	-0.0059	-1.0726e+04	3.0603e+14	4.9976e+04	-8.8021e-04	5.2093e+06
-3.1423e+05	-2.8476e+04	-1.7537e+05	2.3113e+04	-1.1596e+11	2.7780e+04	3.0603e+22	-8.6715e+03	5.6223e+13
0.0076	-0.0124	-1.3789e-04	0.0207	-1.2281e+03	2.6636e-04	9.0431e+03	3.0603e+14	6.1184e+05
-7.4817e+07	-3.8779e+06	-3.6945e+07	-5.9276e+07	-2.5416e+13	6.1884e+06	1.1191e+14	-3.0806e+06	3.0603e+22

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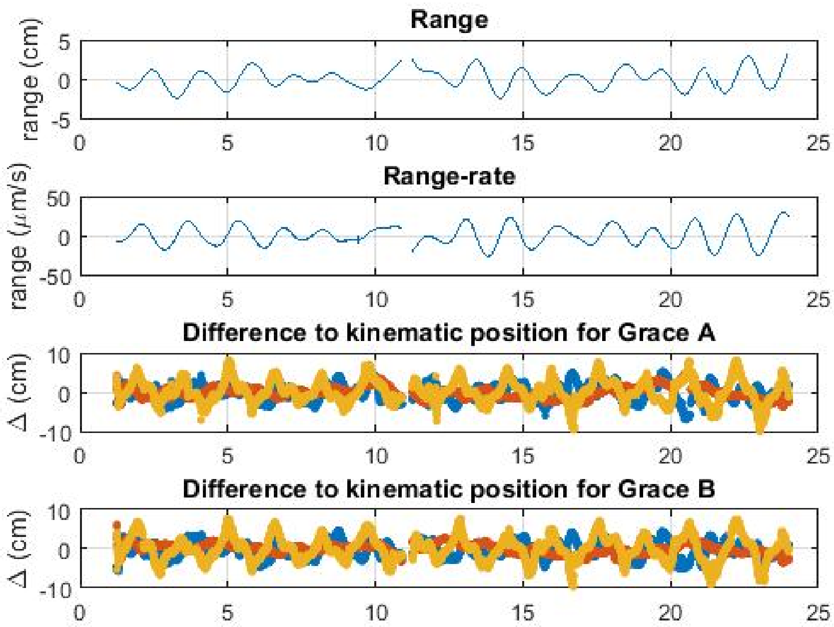
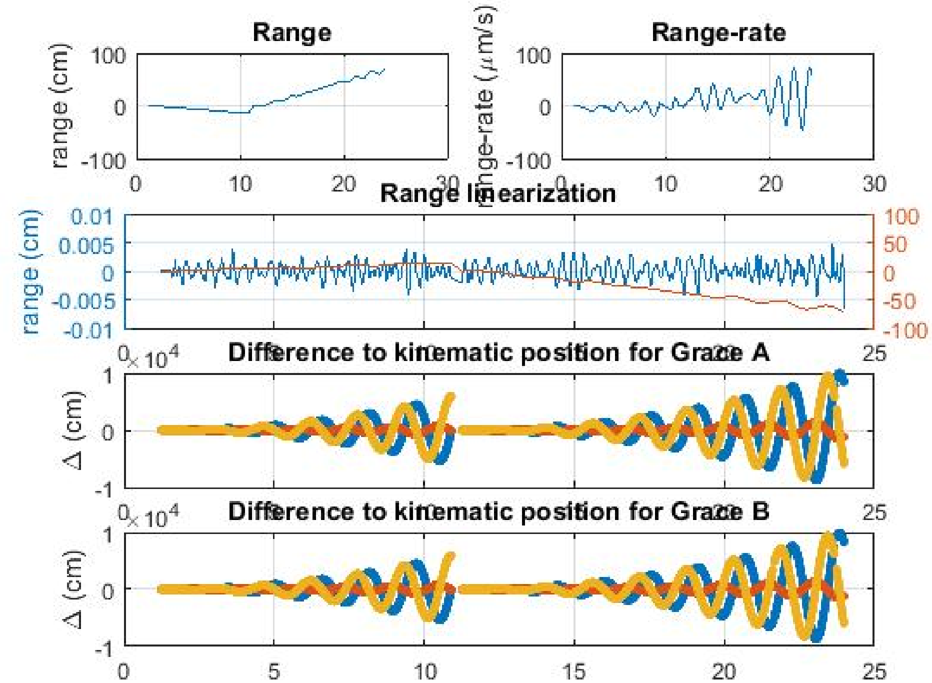
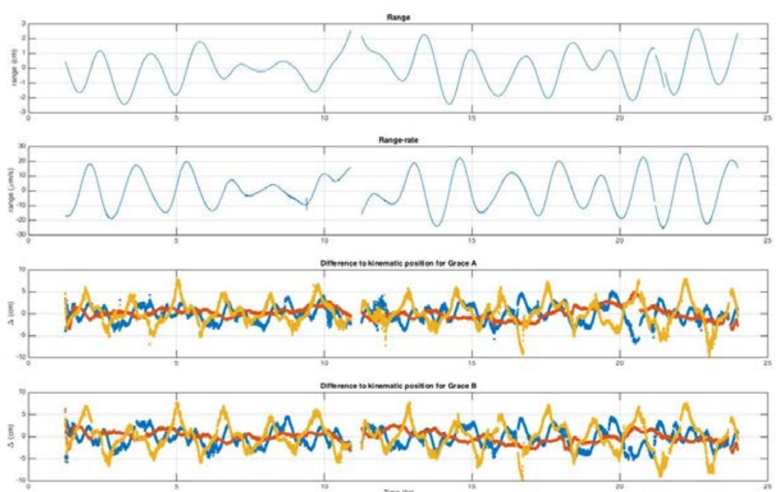
Comparison



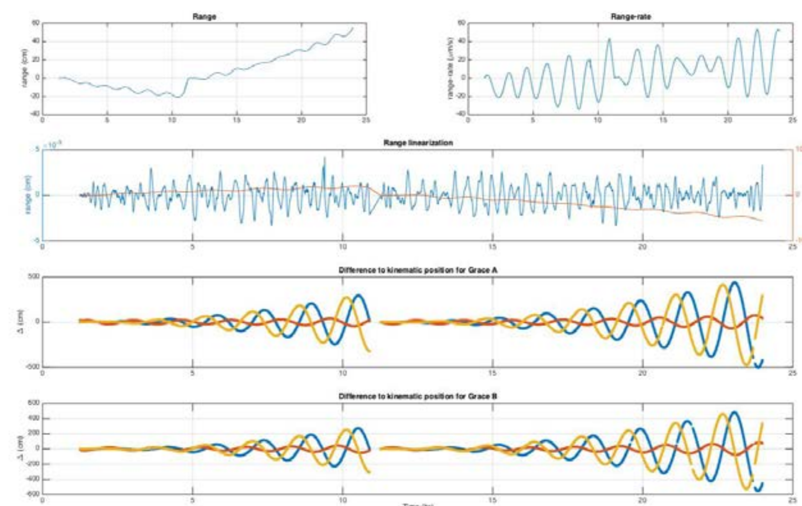
Range
range (cm)
Range-rate
range (μm/s)
Difference to kinematic position for Grace A
Δ (cm)
Difference to kinematic position for Grace B
Δ (cm)



Range
range (cm)
Range-rate
range-rate (μm/s)
Range linearization
range (cm)
Difference to kinematic position for Grace A
Δ (cm)
Difference to kinematic position for Grace B
Δ (cm)

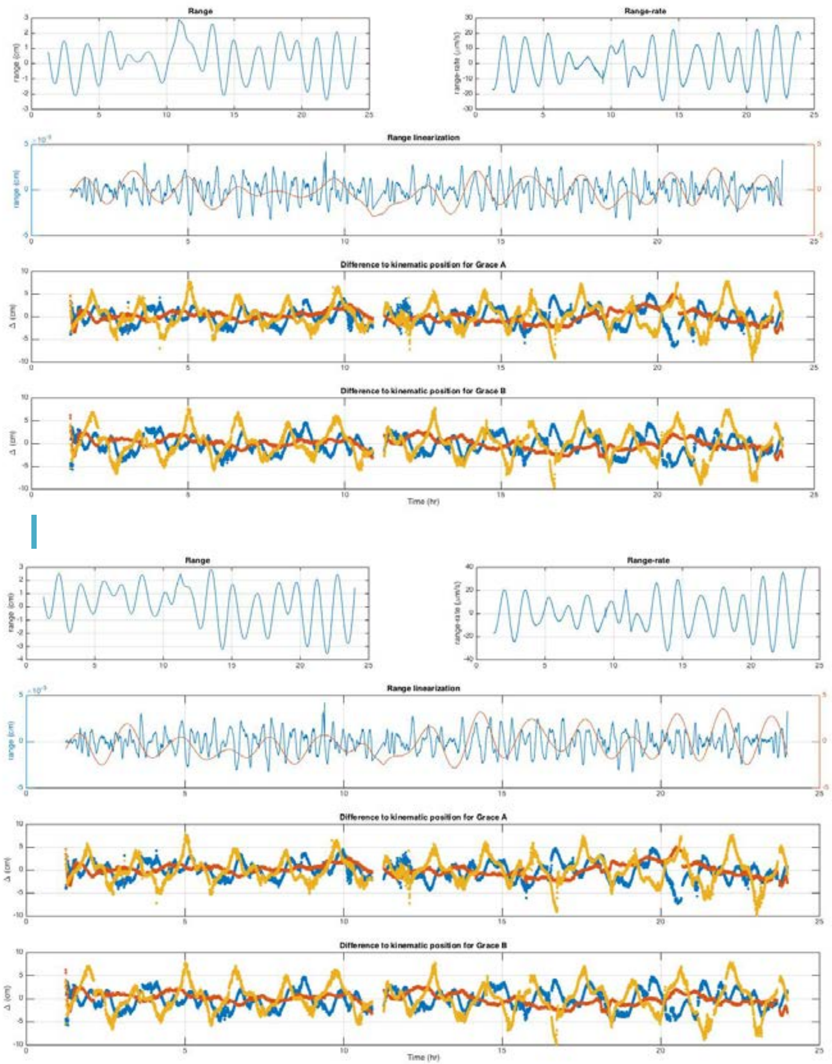


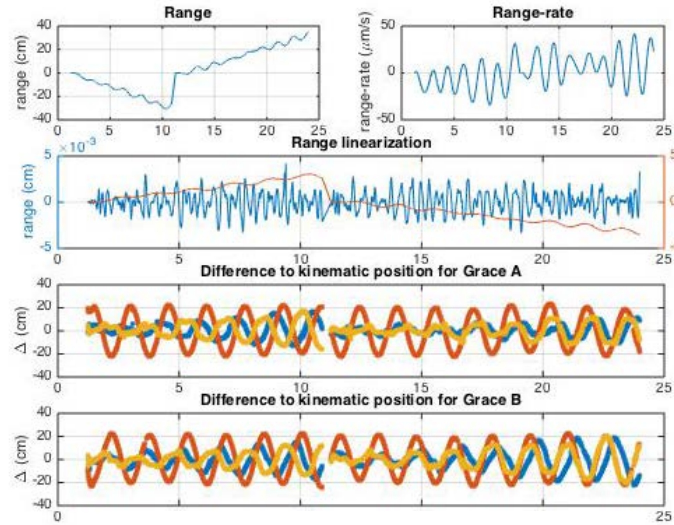
Range
Range-rate
Difference to kinematic position for Grace A
Difference to kinematic position for Grace B
Time (hr)

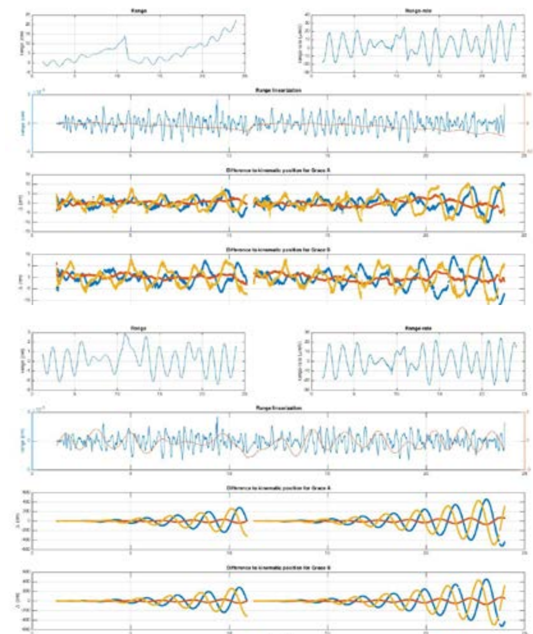


Range
Range-rate
Range linearization
Difference to kinematic position for Grace A
Difference to kinematic position for Grace B
Time (hr)

Comparison







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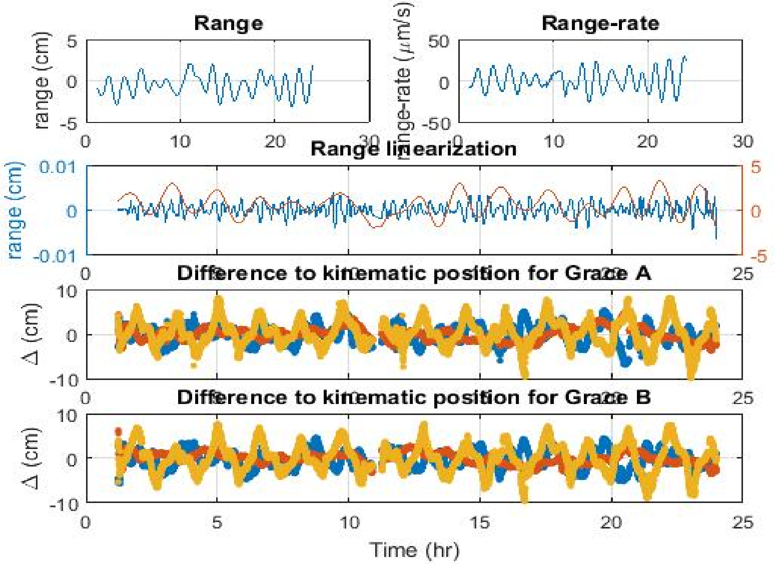
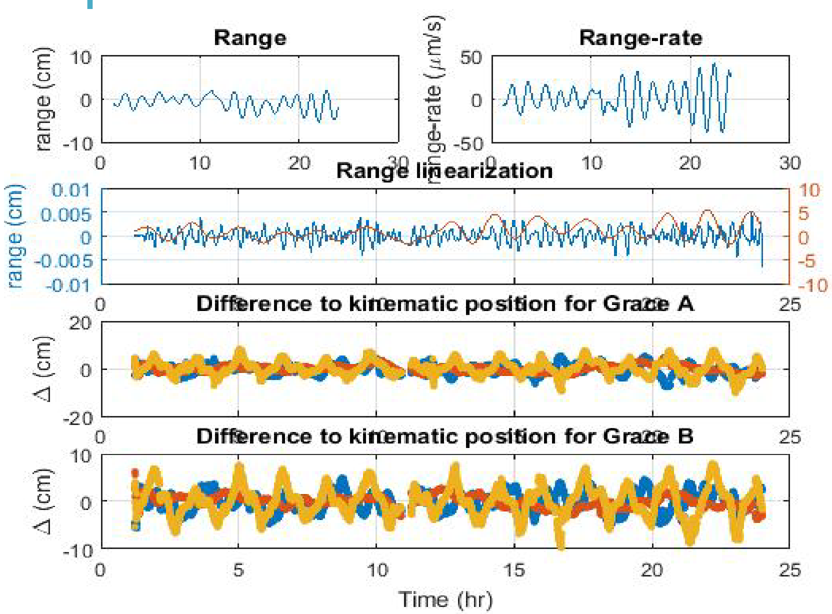
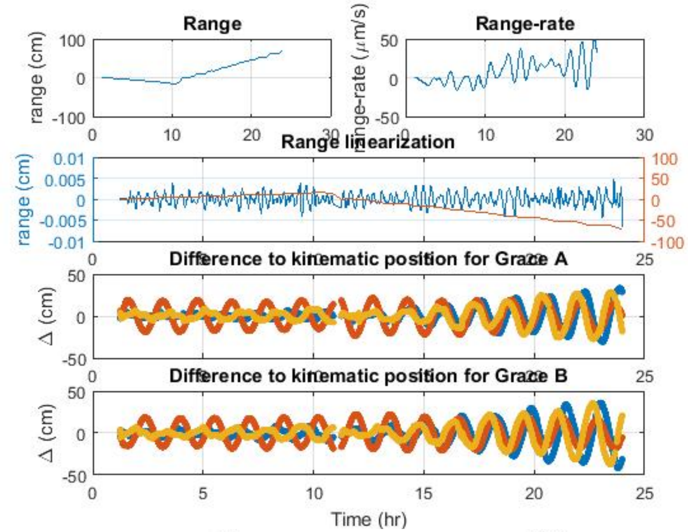
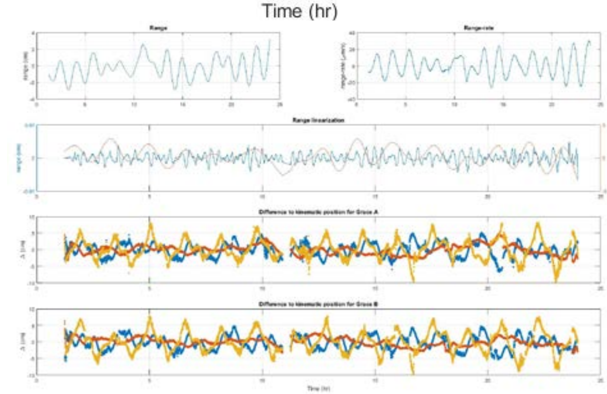
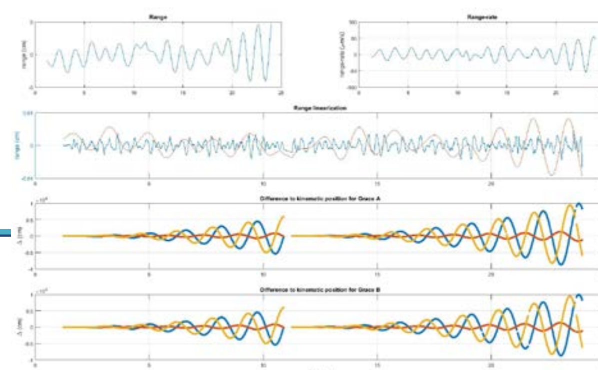
Comparison











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Our strategies

- **Orbit adjustment:**
  - **Common parameter**
    - **Acc scale + bias**
  - **Arc specific parameter**
    - **Empirical linear acc (15min), initial conditions (no constrains)**
- **Combined adjustment:**
  - **Common parameter**
    - **Range bias + Acc scale + bias (9)**
  - **Arc specific parameter**
    - **Empirical linear acc (15min), initial conditions**
- **Constraining values:**
  - **Empirical linear acc: 5e-9**
  - **Acc scale: 1e-4**
  - **Acc bias: 1e-8**
  - **Pos: 1e-1**
  - **Vel: 1e-2**
  - **Scaling factor for y and z components (emp acc,, acc scale and bias) : 1e-16**

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