

Combination on Normal Equation Level

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- Why?
- How?
- First results!

Combination on Normal Equation Level

- To fully take into account correlations between gravity field, orbit, instrument and stochastic parameters, solutions have to be combined on normal equation level.
- All NEQs are based on common standards on reference frames, Earth orientation, relativity, and third bodies.
- Processing approaches, parametrization and background models are not harmonized.

Combination on NEQ-level is the special thing about the EGSIM combination service!!!

NEQ-Format: SINEX

The information is stored in the following blocks:

- FILE/REFERENCE
- FILE/COMMENT
- SOLUTION/STATISTICS
- SOLUTION/ESTIMATE
- SOLUTION/APRIORI
- SOLUTION/NORMAL_EQUATION_VECTOR
- SOLUTION/NORMAL_EQUATION_MATRIX

SINEX: COMMENT and STATISTICS

- FILE/COMMENT:

– earth_gravity_constant	3.9860044150e+14
– radius	6.3781363000e+06
– tide_system	zero_tide / tide_free

- SOLUTION/STATISTICS

– NUMBER OF OBSERVATIONS	540481
– NUMBER OF UNKNOWNNS	8277
– NUMBER OF DEGREES OF FREEDOM	532204
– WEIGHTED SQUARE SUM OF O-C	5.1761025e+05

SINEX: Data

- SOLUTION/ESTIMATE

- 1 CN 2 -- 0 06:016:43200 ---- 2 -4.84169160788564e-04 1.39923e-11
- 2 CN 2 -- 1 06:016:43200 ---- 2 -3.41480150232469e-10 8.80419e-12
- 3 SN 2 -- 1 06:016:43200 ---- 2 1.46383672520029e-09 8.37504e-12

- SOLUTION/APRIORI

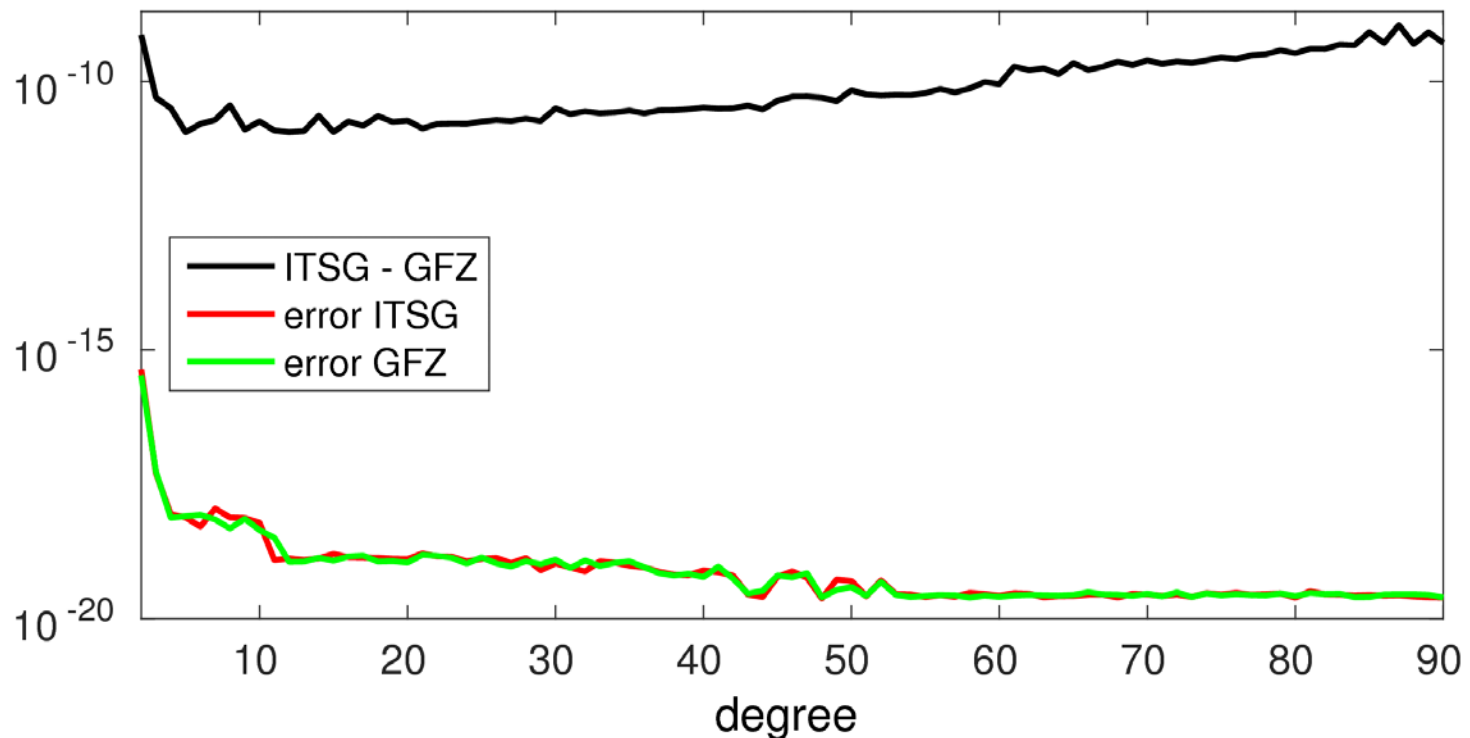
- 1 CN 2 -- 0 06:016:43200 ---- 2 -4.84169219812195e-04
- 2 CN 2 -- 1 06:016:43200 ---- 2 -2.87591948230532e-10
- 3 SN 2 -- 1 06:016:43200 ---- 2 1.47690500410210e-09

- SOLUTION/NORMAL_EQUATION_VECTOR

- 1 CN 2 -- 0 06:016:43200 ---- 2 4.04254781162723e+11
- 2 CN 2 -- 1 06:016:43200 ---- 2 -6.85974043792560e+11
- 3 SN 2 -- 1 06:016:43200 ---- 2 7.71101358350703e+10

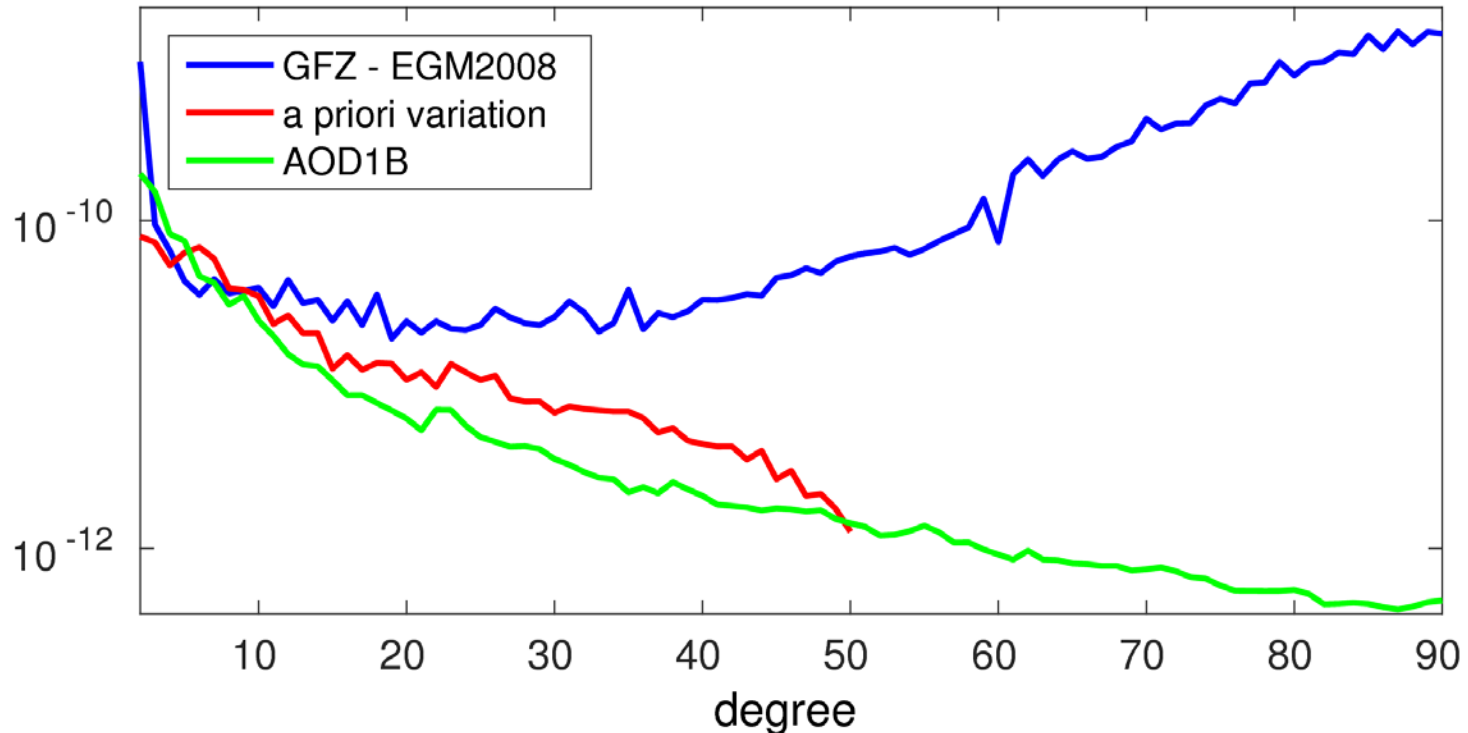
Test of consistency

NEQs are converted from SINEX to NQ0 and inverted by ADDNEQ2. The solution is compared to SOLUTION/ESTIMATE.



A priori values

- SOLUTION/APRIORI contains the a priori static gravity field (plus monthly mean of a priori temporal variations).
- Monthly mean of background or dealiasing models may be added (development during operational service phase).



Observables

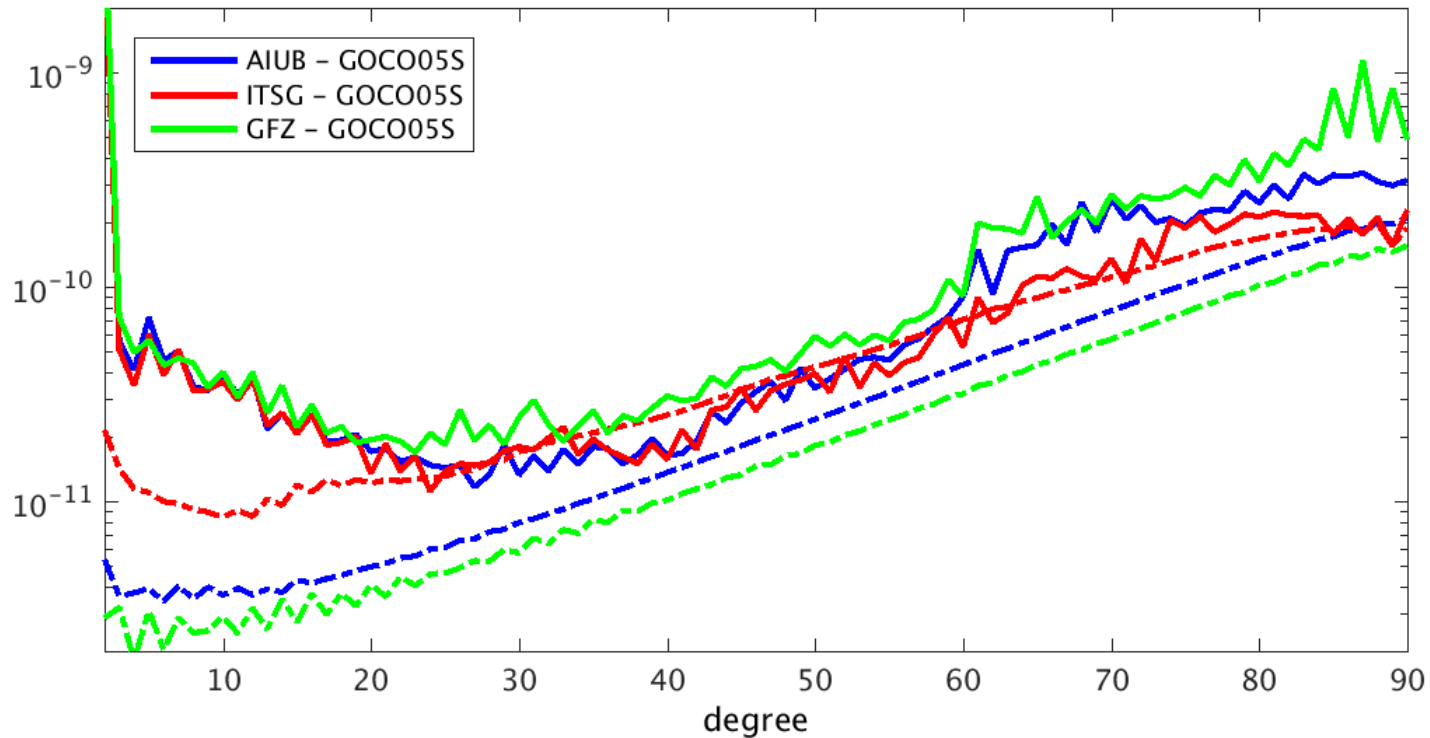
The individual NEQs are based on GPS phases (**GPS**) or kinematic satellite orbits (**POS**), K-band range-rates (**KRR**) (and pseudo-observations of instrument or arc-specific parameters).

f=15	SO	W_{KRR}	W_{GPS}	W_{POS}	W_{norm}
KRR	$SO_{KRR} = 3e-7 \text{ m s}^{-1}$	1	$SO_{GPS}^2 / SO_{KRR}^2 = 1e10$	$SO_{POS}^2 / SO_{KRR}^2 = 1e12$	$1 / SO_{KRR}^2 = 1.11e13$
GPS	$SO_{GPS} = f \cdot 2e-3 \text{ m} = 0.03 \text{ m}$	$SO_{KRR}^2 / SO_{GPS}^2 = 1e-10$	1		$1 / SO_{GPS}^2 = 1111.11$
POS	$SO_{POS} = f \cdot 2e-2 \text{ m} = 0.3 \text{ m}$	$SO_{KRR}^2 / SO_{POS}^2 = 1e-12$		1	$1 / SO_{POS}^2 = 11.11$
STOCH. ACCEL.	$SO_{cons} = 3e-9 \text{ s}^{-2}$	$SO_{KRR}^2 / SO_{cons}^2 = 1e4$	$SO_{GPS}^2 / SO_{cons}^2 = 1e14$	$SO_{POS}^2 / SO_{cons}^2 = 1e16$	$1 / SO_{cons}^2 = 1.11e17$

Observation types, sampling rates and relative weighting of observations may vary. For combination NEQs are normalized.

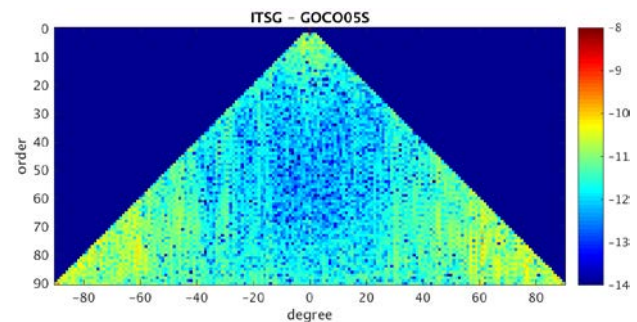
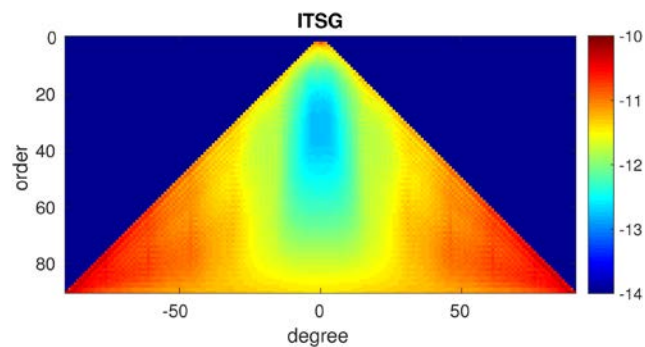
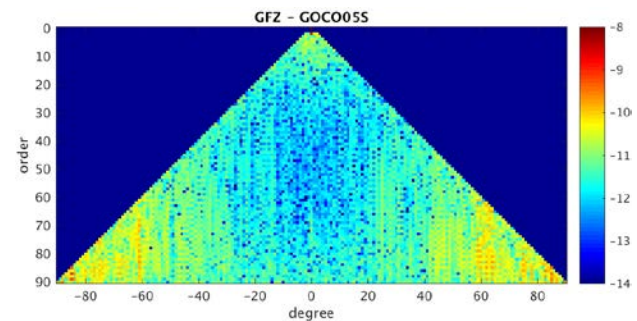
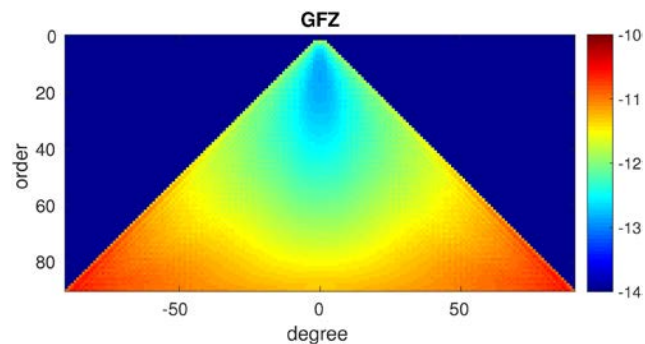
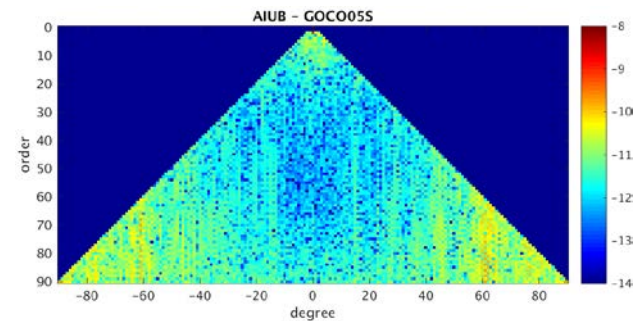
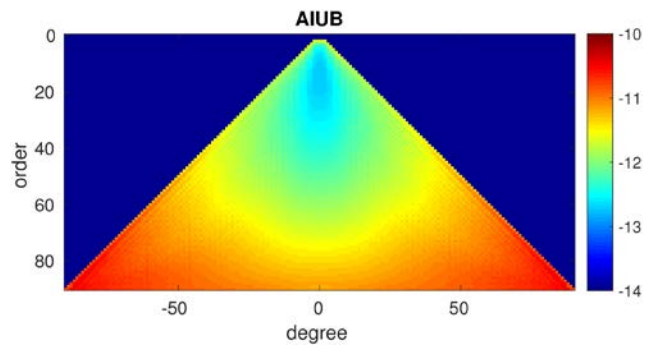
Noise model

Noise models and consequently formal errors vary significantly.

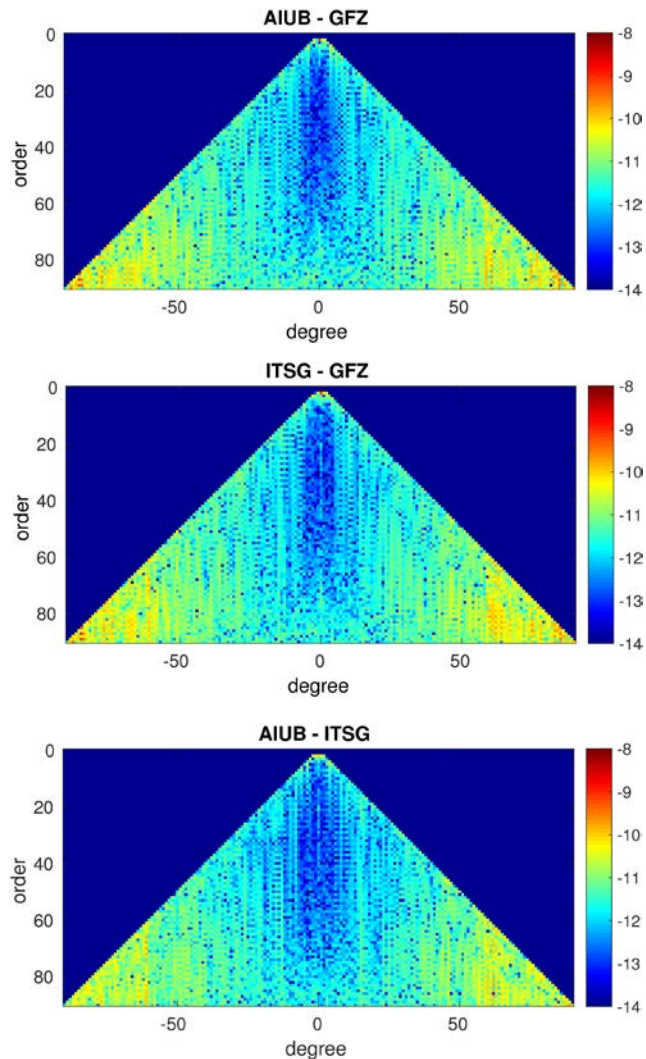


ITSG applies empirical co-variances, leading to realistic formal errors.

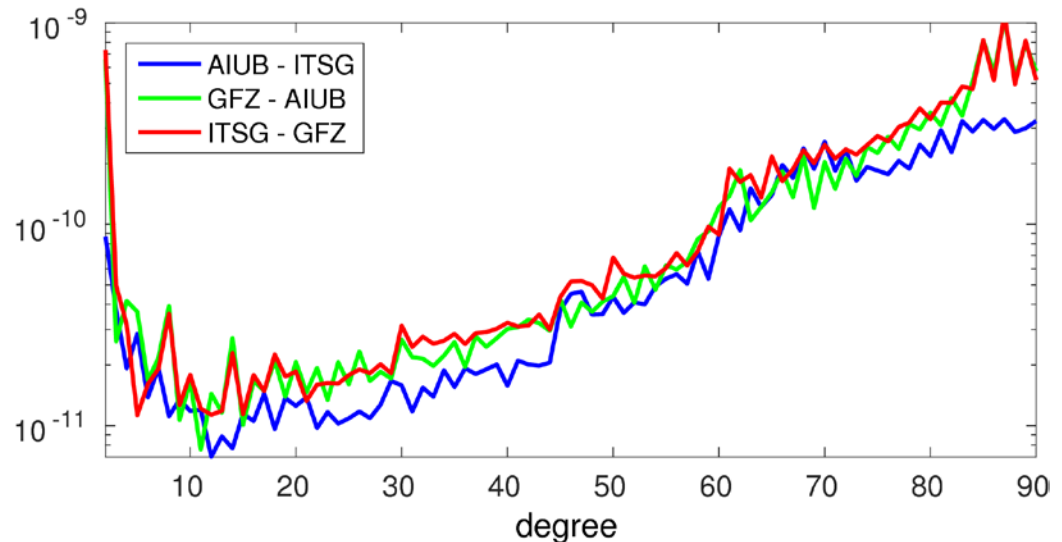
Formal Errors and Differences to GOCO05S



Comparison at solution level



- The consistency between AIUB and ITSG is higher than to GFZ.
- ITSG zonal coefficients differ due to sensor fusion ATT + ACC.



Individual contributions: AIUB, GFZ, ITSG

Observables:

- AIUB: 1016763 (POS at 30s, KRR at 5s)
- GFZ: 2691802 (GPS at 30s, KRR at 5s)
- ITSG: 540481 (POS at 300 s, KRR at 5s)

Parameters:

- 8277 (gravity field coefficients of degrees 2 to 90)

All orbit, instrument or stochastic parameters are pre-eliminated **(and statistics corrected accordingly)**.

Degree 1 terms have to be handled consistently. They may be set to zero / fixed at their specific a priori values (**0**) by AIUB.

Relative weights from NEQ statistics

Relative weights are based on a posteriori RMS:

$$W = S_0^2 / \text{RMS}^2 = 1 / \text{RMS}^2 \quad (\text{in case of normalization})$$

$$\text{RMS}^2 = \mathbf{v}^T \mathbf{P} \mathbf{v} / \text{DOF}$$

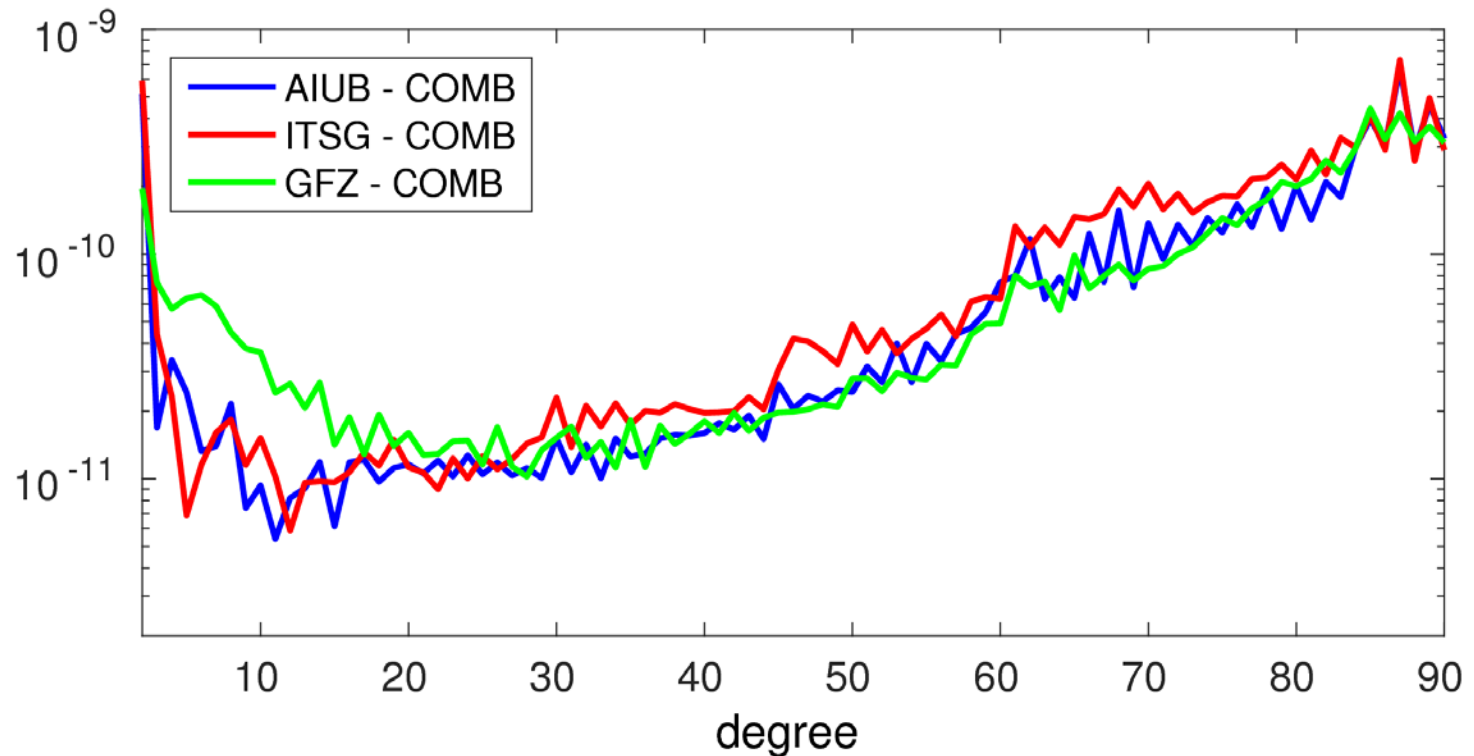
$\text{DOF} = n_{\text{obs}} - n_{\text{par}}$ (corrected for pseudo-observations / pre-eliminated parameters)

$\mathbf{v}^T \mathbf{P} \mathbf{v} = \mathbf{l}^T \mathbf{P} \mathbf{l} - \mathbf{d} \mathbf{x}^T \mathbf{b}$ with \mathbf{v} = residuals, \mathbf{l} = observations, \mathbf{P} = weights,
 $\mathbf{d} \mathbf{x}$ = ESTIMATE – APRIORI, \mathbf{b} = NORMAL_EQUATION_VECTOR

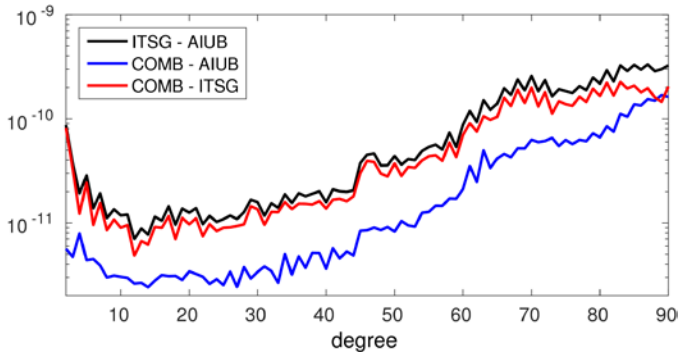
$S_0 = 1$	DOF	$\mathbf{l}^T \mathbf{P} \mathbf{l}$	$\mathbf{v}^T \mathbf{P} \mathbf{v}$	RMS	S_0^2 / RMS^2
AIUB	1008486	178615	161893	0.40	6.25
GFZ	2683525	2599539	2065152	0.88	1.30
ITSG	532204	517610	495045	0.96	1.08

Combination: AIUB + GFZ + ITSG

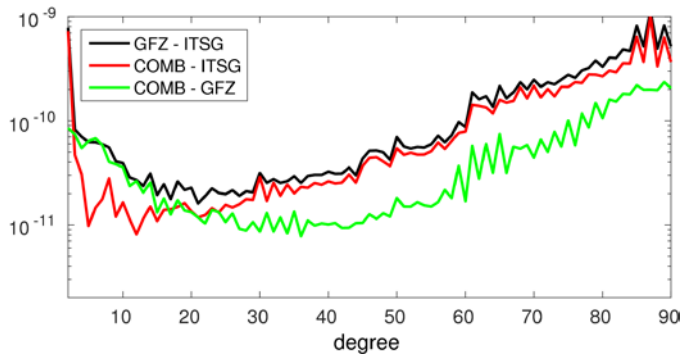
- A combination based on NEQ-statistics leads to a down-weighting of ITSG relative to AIUB and GFZ.
- GFZ contributes less to low degree coefficients.



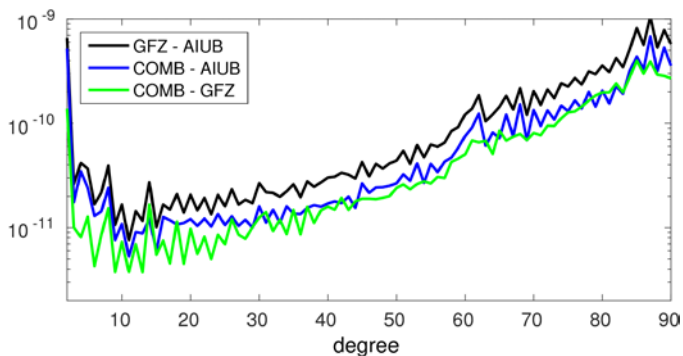
Combination: individual contributions



AIUB and ITSG contributions run parallel, but ITSG is punished for realistic error levels.



GFZ contributes little at low degrees to combination with ITSG, but dominates middle to high degrees.

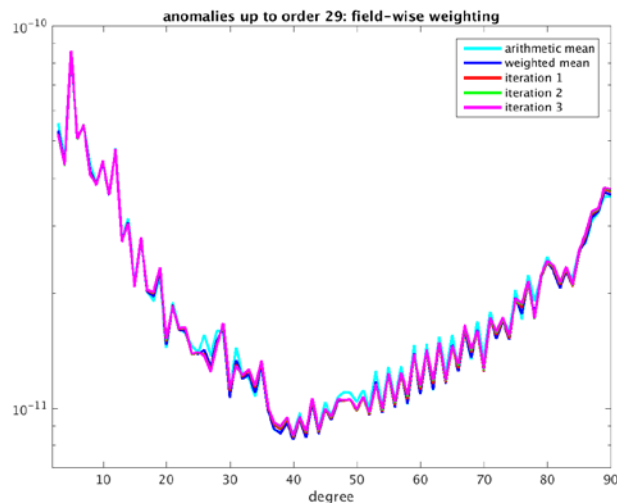
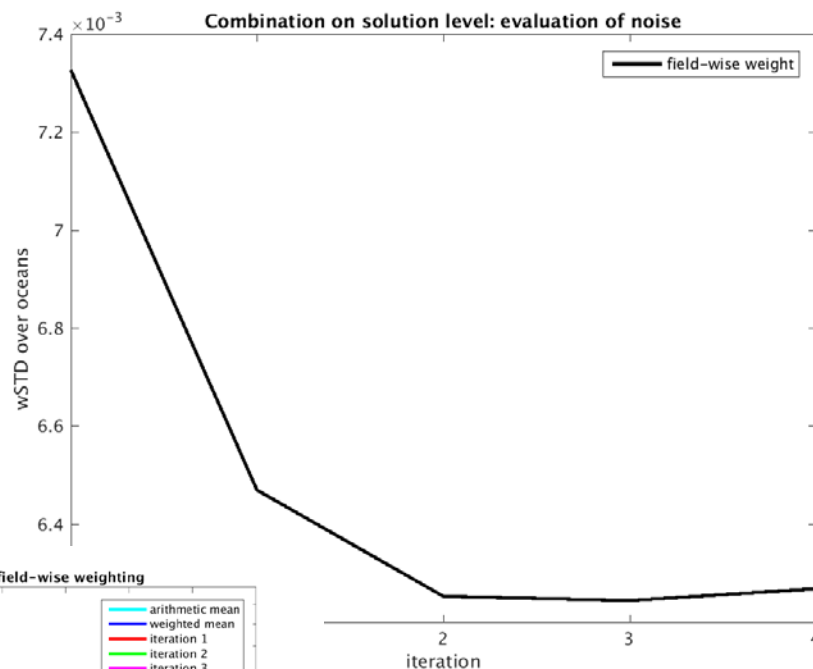
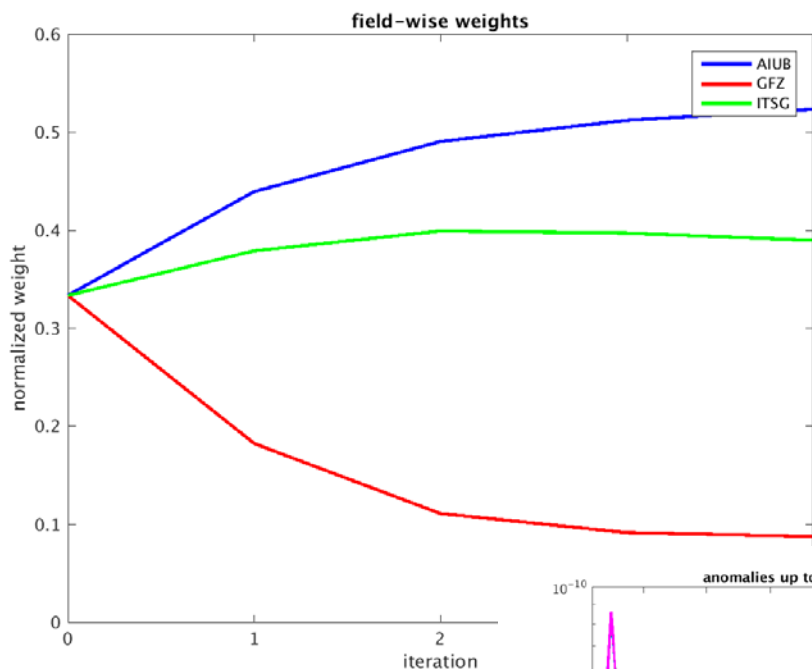


Weights perform best for AIUB + GFZ combination, individual contributions correspond to relative levels of formal errors.

Combination Results

- Solution level
- NEQ level

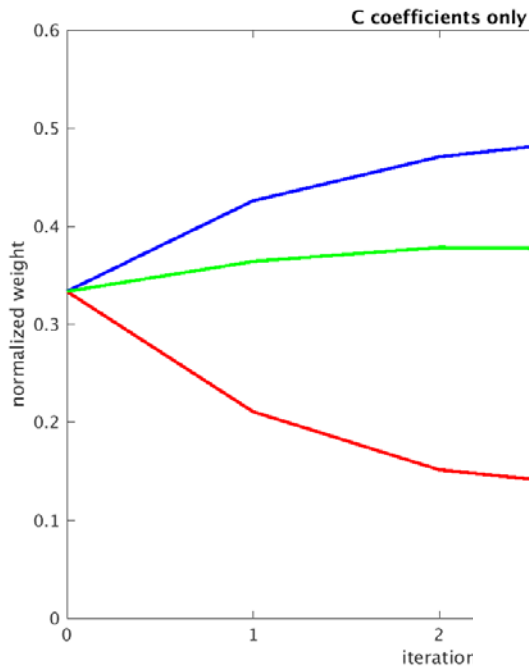
Combination on solution level: weights (VCE)



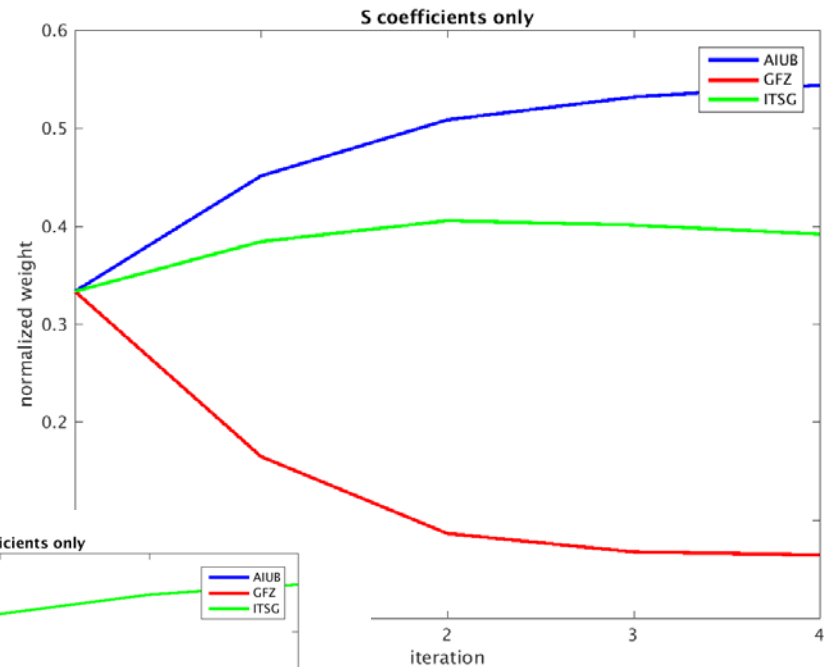
wSTD oceans:

- AIUB: $8.2e-3$
- GFZ: $14.4e-3$
- ITSG: $5.5e-3$

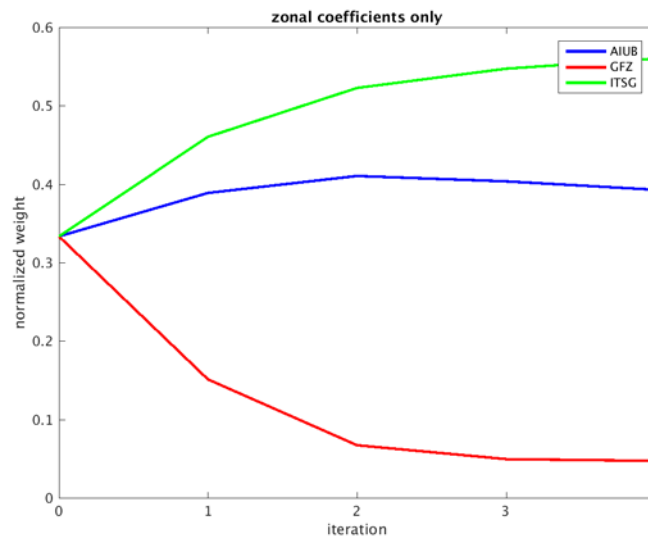
Are the weights characteristic for whole spectrum?



4094 coefficients



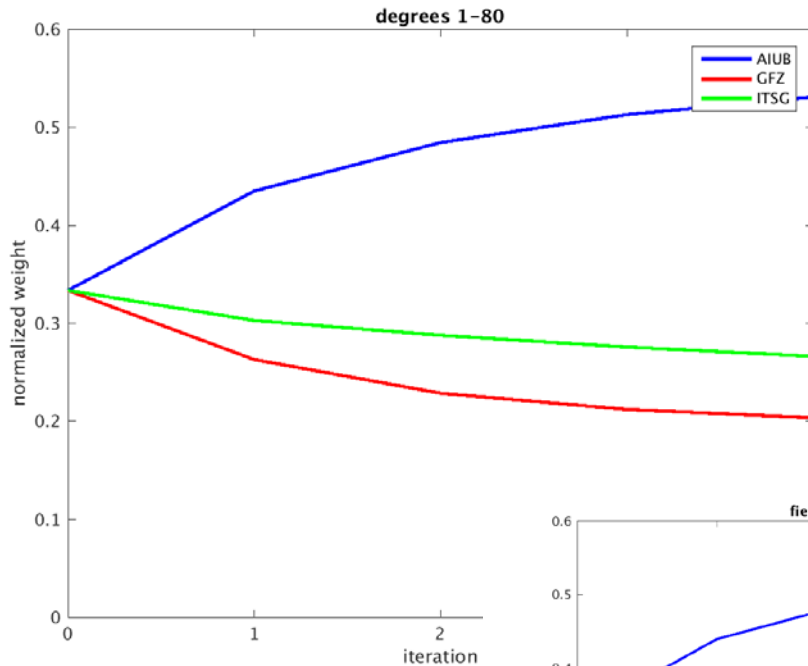
4094 coefficients



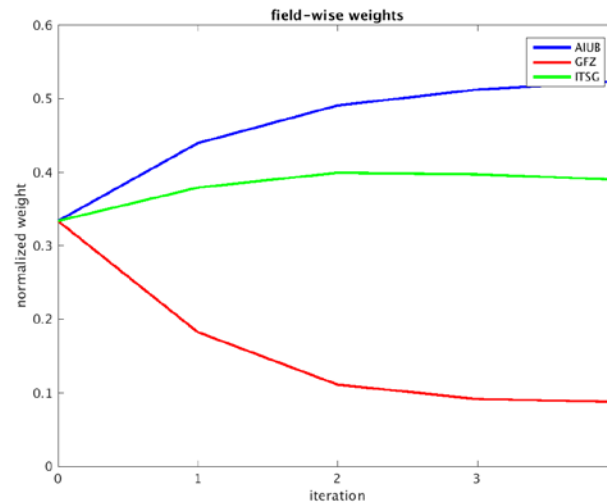
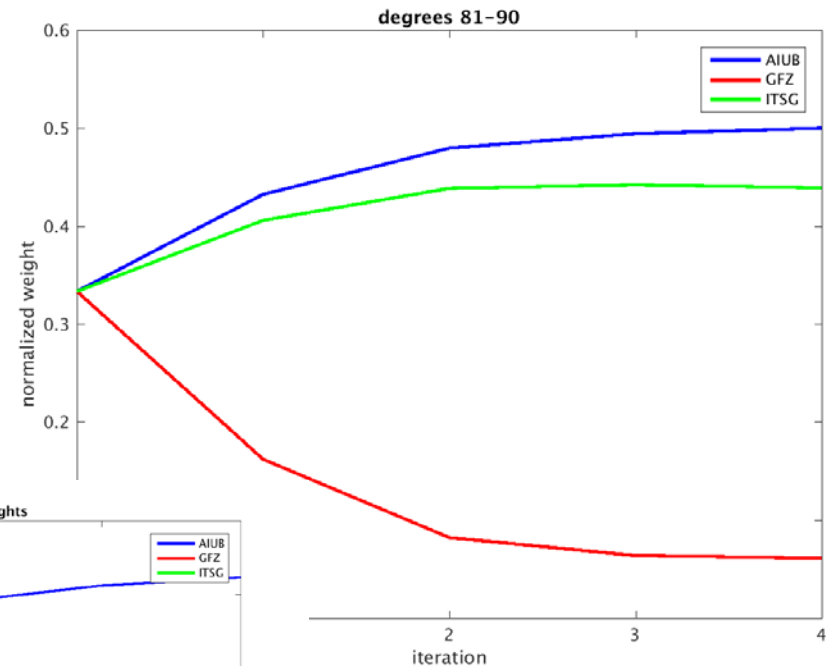
89 coefficients

Are the weights characteristic for whole spectrum?

6557 coefficients

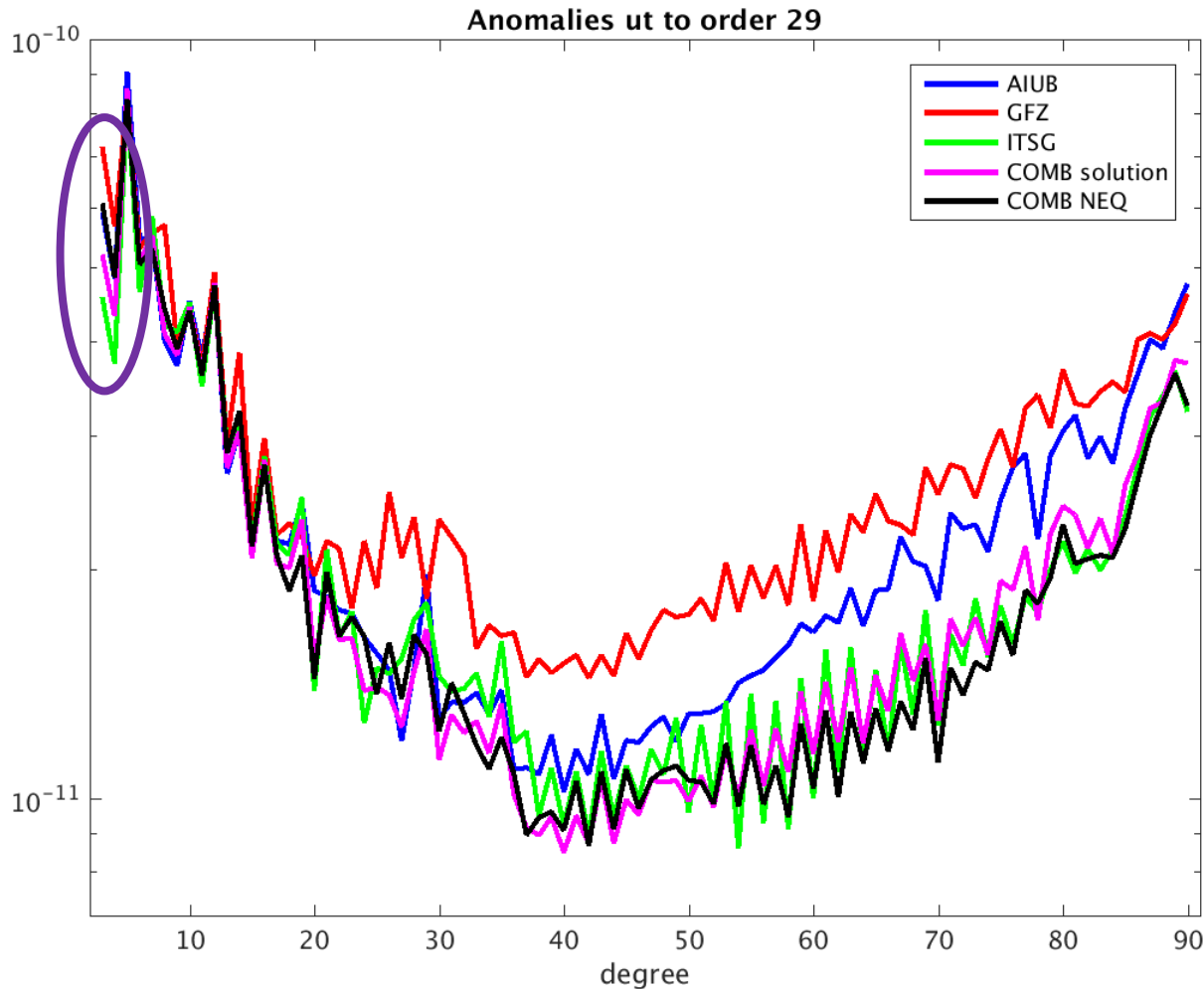


1720 coefficients



All (8277) coefficients

Combination on NEQ level: weights from solutions

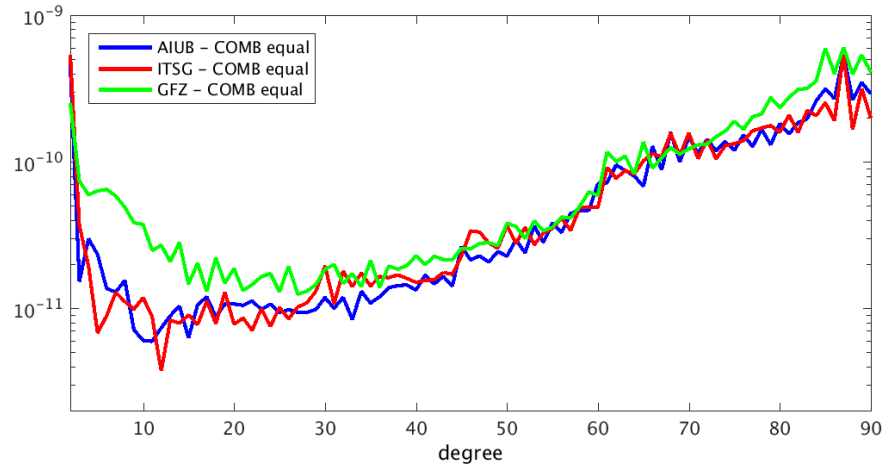
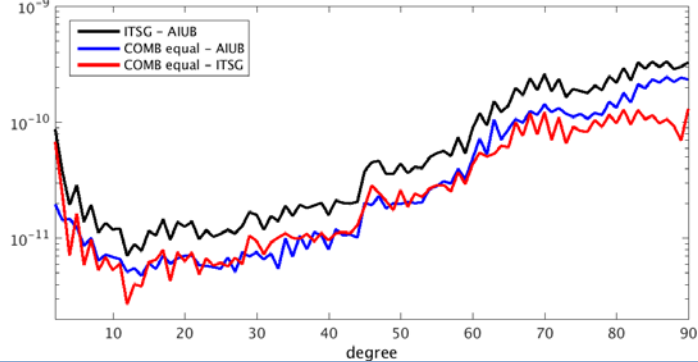
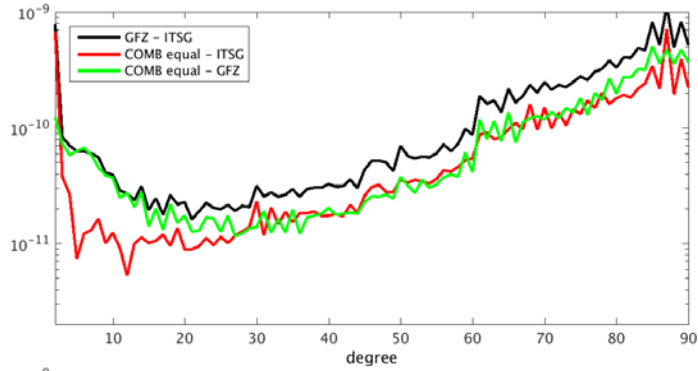
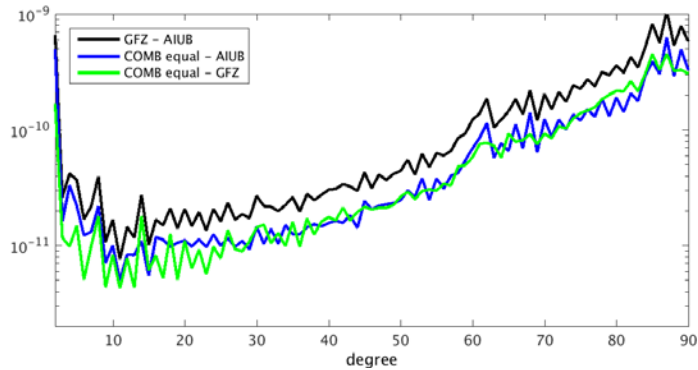


wSTD oceans:

- AIUB: $8.2e-3$
- GFZ: $14.4e-3$
- ITSG: $5.5e-3$

- SOL: $6.3e-3$
- NEQ: $7.7e-3$

Equal contribution by empirical weighting



Empirical * solution derived relative weights

Equal contribution is approx. reached for relative weights of

- AIUB: 6.25
- GFZ: 1 (instead of 1.30)
- ITSG: 5 (instead of 1.08)

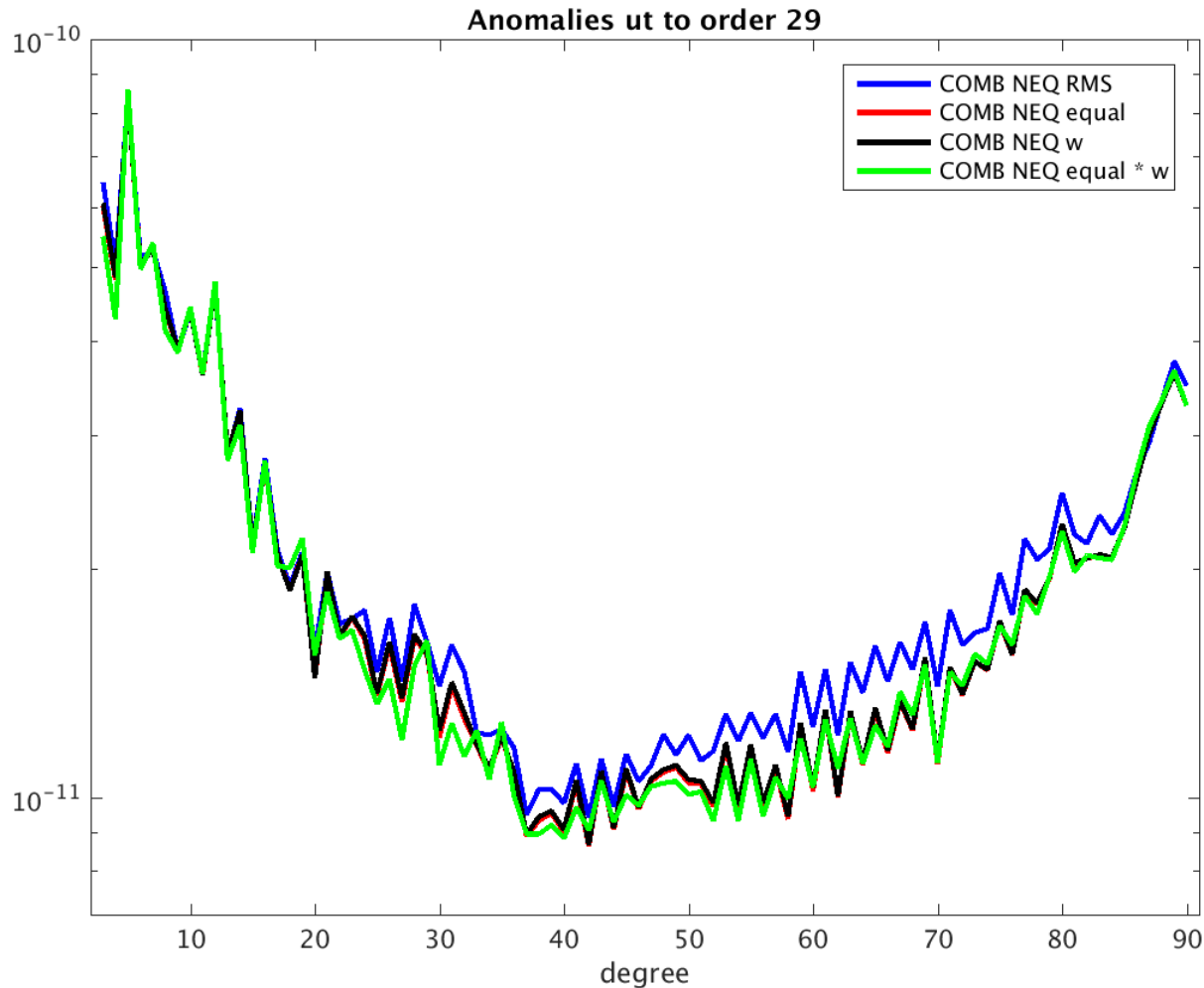
Comparison at solution level leads to

- AIUB: $0.51 \triangleq 5.67$
- GFZ: $0.09 \triangleq 1$
- ITSG: $0.40 \triangleq 4.44$

Weighting corresponding to solution level is reached by

- AIUB: $6.25 * 5.67 = 35.44$
- GFZ: 1
- ITSG: $5 * 4.44 = 22.20$

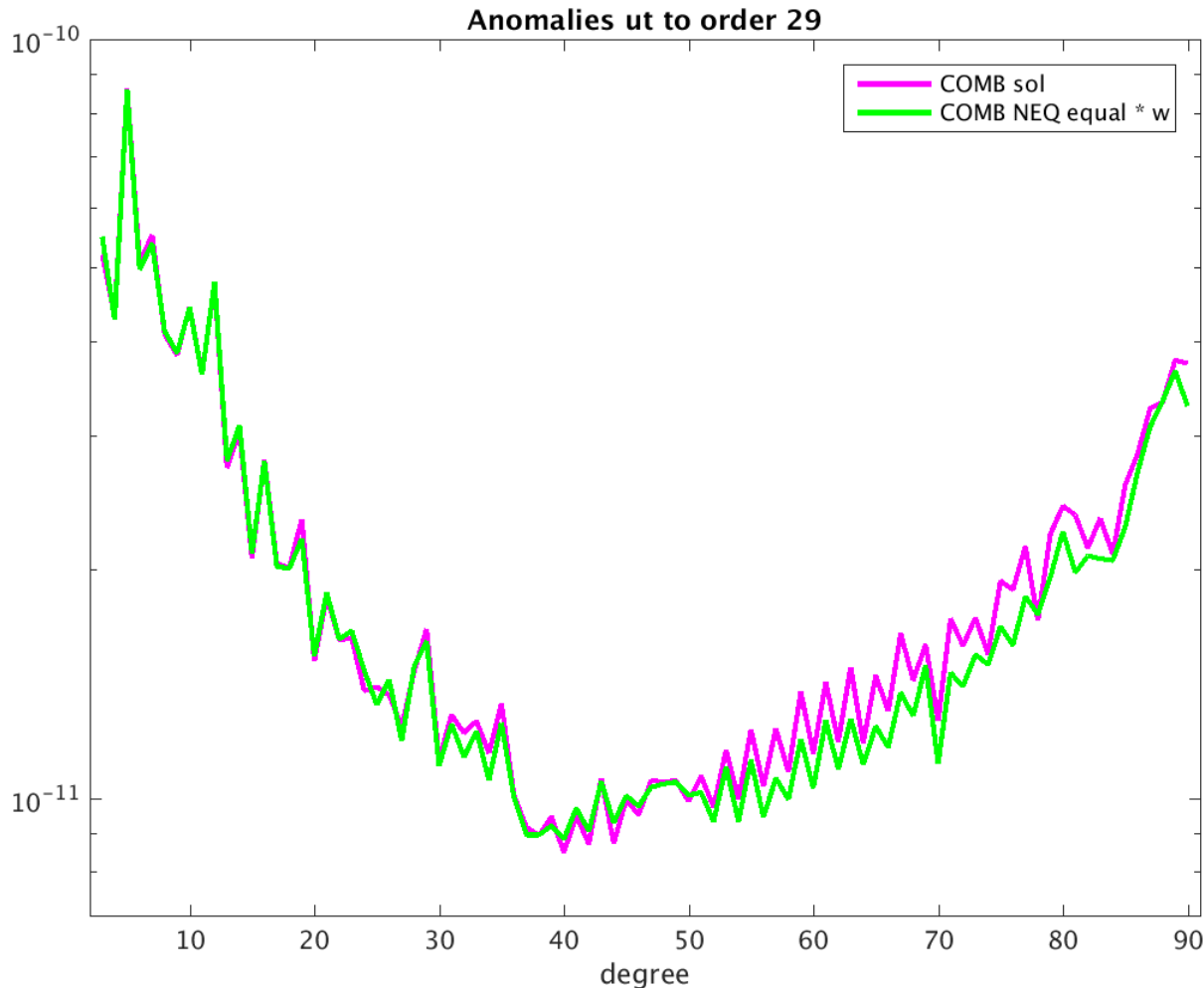
Combination on NEQ level



wSTD oceans:

- RMS: $9.5e-3$
- equal: $7.5e-3$
- equal*w: $5.9e-3$

Comparison NEQ / SOL - combination



wSTD oceans:

- SOL: $6.3e-3$
- NEQ: $5.9e-3$

Signal-dominated part is consistent.

Noise-dominated part is better!

Conclusions

- It's working!
- Automated process to reach comparable contribution of individual NEQs is needed.
- Contribution analysis
- Scaling of NEQs to common R is still missing (effect mainly on degree 2).
- Format transformation of NEQs Bernese – SINEX is still missing.